IVT Applications and Limit Definition of Derivative

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

1. Use the Intermediate Value Theorem to find points of intersection on the graphs of \( y = x^3 - 3x^2 \) and \( y = x - 5 \).
   This is equivalent to finding x-values for the roots of \( f(x) = x^3 - 3x^2 - x + 5 \).

2. Show that there is a number whose third power is equal to the sum of its lower powers.
   (Hint: Rewrite the equation so that it is equal to 0, then find an interval where one endpoint has positive height and the other endpoint has negative height. Now apply the Intermediate Value Theorem.)
   \[ x^3 = 1 + x + x^2 \]
3. Show that there exists a rectangular box with edges $x, x + 1$ and $x + 2$ for some positive $x$ so that the volume of the box is equal to its surface area (the box has both a top and a bottom).

(Hint: Draw a picture of the box and write equations that describe its volume and surface area. Set these equal to each other, and continue on as in Problem 2.)

4. Using the limit definition of derivative, find the derivative function, $f'(x)$, of the following function. Show all work.

$$f(x) = -x^2 + 2x$$

5. The following are the graphs of the position of two racers.

Which racer has a faster average speed? When was each racer running faster than the other (approximately)? Which racer was running the fastest at the end of the race? Explain your answers.