CONTINUITY, LIMITS AT INFINITY

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

1. Sketch a graph of the functions below, and fill in the following limits at infinity:

\[
ed^x \quad e^{-x} \quad \ln(x) \quad -\ln(x) \quad \sin(x) \quad \arctan(x)\]

(a) \( \lim_{x \to +\infty} e^x = \)  
(b) \( \lim_{x \to -\infty} e^x = \)  
(c) \( \lim_{x \to +\infty} e^{-x} = \)  
(d) \( \lim_{x \to -\infty} e^{-x} = \)  
(e) \( \lim_{x \to +\infty} \ln(x) = \)  
(f) \( \lim_{x \to -\infty} \ln(x) = \)  
(g) \( \lim_{x \to +\infty} -\ln(x) = \)  
(h) \( \lim_{x \to -\infty} -\ln(x) = \)  
(i) \( \lim_{x \to +\infty} \sin(x) = \)  
(j) \( \lim_{x \to -\infty} \sin(x) = \)  
(k) \( \lim_{x \to +\infty} \arctan(x) = \)  
(l) \( \lim_{x \to -\infty} \arctan(x) = \)

Example.

An algebraic technique you can use to find a limit at infinity is to factor out the highest power you see in the denominator, from both the numerator and denominator.

\[
\lim_{x \to \infty} \frac{8x + 6}{3x - 1} = \lim_{x \to \infty} \frac{8 + 6/x}{3 - 1/x} \times \frac{x}{x} = \lim_{x \to \infty} \frac{8 + 6/x}{3 - 1/x} = \frac{8}{3}
\]

2. \( \lim_{x \to -\infty} \frac{\sqrt{2x^2 + 1}}{3x - 5} \)

3. \( \lim_{x \to \infty} \left( \sqrt{x^2 + 1} - x \right) \)

4. \( \lim_{x \to +\infty} \frac{x^2 + x}{3 - x} \)
5. Determine the equation for each horizontal asymptote on the graph of the following function. Your answer must be justified with limits.

\[ f(x) = \frac{36 + 42e^{6x}}{2e^{6x} - 46} \]

6. Find the values of \( a \) and \( b \) that make \( f \) continuous everywhere

\[ f(x) = \begin{cases} 
\frac{x^2 - 4}{x - 2} & \text{if } x < 2 \\
ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\
2x - a + b & \text{if } x \geq 3
\end{cases} \]

7. Fill in the blanks: The Intermediate Value Theorem says that

IF

THEN

8. Use the Squeeze Theorem to find \( \lim_{x \to +\infty} \frac{\cos(x^2 \sin(x))}{e^x} \)