(1) Give the definition of $f'(x)$. Let $f(x) = 5x^2 - 8x$.

Use the definition of a derivative as a limit to prove that $f'(x) = 10x - 8$.

Show each step in your calculation and be sure to use proper terminology in each step of your proof.

(2) A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 450 cells. Find an expression, $P(t)$, for the number of bacteria after $t$ hours.

(3) The slope of the tangent line at each point on the graph of $y = f(x)$ is equal to eight times the value of its y-coordinate. Given that $f(9) = 3$, determine a formula for $f(x)$. 
(4) Find the $x$-value for each point on the graph of $g(x) = x^3 + 10x + 42$ where the line tangent to the curve is perpendicular to the line $x + 25y = 100$.

(5) Iodine-131 has a half-life of 8 days. What is the original mass in $mg$ of a sample if it has decayed to a mass of 150 $mg$ after 21 days?

(6) Determine an equation for each horizontal asymptote on the graph of the function.

$$f(x) = \begin{cases} 
36 \arctan(x) + 60\pi & \text{if } x < 0 \\
60e^{-3x} - 120 & \text{if } x \geq 0 
\end{cases}$$

(7) Find the equation of the line tangent to the curve at its positive $x$-intercept.

$$x^5y + 25 = x^2 + y^{12}$$
The position in meters of a particle at time $t \geq 0$ seconds is given by

$$s(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 8t + 90$$

What is the particle’s acceleration at the moment when the particle’s velocity is 20 m/s? Use correct units in your final answer.

The function $f(x) = 5e^{4x} - 50$ has derivative $f'(x) = 20e^{4x}$. Determine the equation for the line which is tangent to the graph of $f(x)$ at its $x$-intercept.

The graphs of $f(x) = \ln(2) + 3\ln(-x)$ and $g(x) = \ln(-32x)$ intersect. Determine the $x$-value for each point of intersection. Simplify your answer.
(11) It’s been awhile since we did limits. Can you evaluate these?

(a) \[
\lim_{x \to \pi^-} \ln(\sin x)
\]

(b) \[
\lim_{x \to 3} \frac{\sqrt{x + 6} - x}{x^3 - 3x^2}
\]

(c) \[
\lim_{x \to \infty} x \tan(1/x)
\]

(d) \[
\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{bx}
\]
(e) \[ \lim_{x\to\infty} \frac{91\sqrt{x} + 3}{5 - 7\sqrt{x}} \]

(f) \[ \lim_{x\to2^+} \frac{e^{3\ln(x)} - 2}{16 - e^{4\ln(x)}} \]

(g) \[ \lim_{x\to3^+} \frac{\ln(10 + x^2)}{\ln(10 - x^2)} \]

(h) \[ \lim_{x\to6^+} \frac{\cos\left(\frac{5\pi}{x}\right)}{\ln(7 - x)} \]

(i) \[ \lim_{x\to-\infty} \frac{10\arctan(8x) + 19\pi}{18\arctan(4x) + 16\pi} \]

(j) \[ \lim_{x\to\infty} \frac{6e^{2x} + 4}{5e^x - 2e^{2x}} \]
(12) It’s been awhile since we did derivatives. Can you evaluate these?

(a) Compute \( \frac{dy}{dx} \) for the given function. Write your answer completely in terms of \( x \).

\[
y = (x^4 + 8)^5
\]

(b) If \( f(x) = x^{42} \arcsin x \), then find \( f'(x) \).

(c) If \( g(t) = \csc \left( \frac{\ln(t)}{t^8 + 21t^2} \right) \), then find \( g'(t) \).

(d) If \( H(w) = \tan^8 \left( \sqrt{e^{9w} + 4} \right) \), then find \( H'(w) \).