**Instructions.** Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

1. Give the definition of **continuous at** \(a\). Give an example of a function which is continuous at all \(x\). Give an example of a function which has a removable discontinuity at \(x = 1\). Give an example of a function which has a jump discontinuity at \(x = 2\).

\[
f(x) \text{ is continuous at } a \quad \text{if} \quad \lim_{x \to a} f(x) = f(a)
\]

\[f(x) = x^2 \quad f(x) = \frac{x-1}{x-1} = \begin{cases} x+1 & x \leq 2 \\ 2x+6 & x > 2 \end{cases}
\]

2. Give the definition of \(f'(x)\). Use the definition to show that \(f'(x) = 3x^2\) for \(f(x) = x^3\).

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}
\]

\[
= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}
\]

\[
= \lim_{h \to 0} \frac{(3x^2 + 3xh + h^2)}{h} = 3x^2
\]
(3) This question concerns the definite integral

$$\int_{2}^{6} 2 - x \, dx$$

(a) Sketch the (net) area represented by the integral and compute it using geometry.
(b) Compute the integral using the fundamental theorem of calculus.
(c) Compute the integral by using the definition of definite integral.

\[
\text{Area a} = \frac{1}{2} (4)(4) = 8
\]

\[
\int_{2}^{6} (2-x) \, dx = 2x - \frac{1}{2} x^2 \bigg|_{2}^{6} = \left[ 2(6) - \frac{1}{2} (6)^2 \right] - \left[ 2(2) - \frac{1}{2} (2)^2 \right] = 12 - 18 - 4 + 2 = -8
\]

\[
\frac{\Delta x}{n} = \frac{6-2}{n} = \frac{4}{n}
\]

\[
\Delta x = \frac{4}{n}
\]

\[
\lim_{n \to \infty} R_n = \lim_{n \to \infty} -8 - \frac{8}{n} = -8
\]

(4) Find a number \( b \) so that the line \( y = b \) divides the region bounded by the curves \( y = x^2 \) and \( y = 4 \) into 2 regions with equal area.

\[
\text{Area}_{\text{left}} = \int_{0}^{b} \sqrt{y} - (-\sqrt{y}) \, dy = \int_{0}^{\sqrt{b}} (\sqrt{y} + \sqrt{y}) \, dy
\]

\[
\phi \text{ a}_{\text{left}} \int_{0}^{b} \sqrt{y} \, dy = \phi \text{ a}_{\text{right}} \int_{0}^{b} \sqrt{y} \, dy
\]

\[
\text{Area}_{\text{left}} = \frac{2}{3} b^{3/2} - 0 = \frac{2}{3} b^{3/2} - 0
\]

\[
\frac{4}{3} (b^{3/2}) - \frac{4}{3} b^{3/2} = \frac{4}{3} b^{3/2} - 0
\]

\[
b = 4^{1/3} = \sqrt[3]{16}
\]

\[
y = 3 \sqrt{16}
\]