NET CHANGE THEOREM AND SUBSTITUTION

**Instructions.** Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

(1) Compute the indefinite integrals.

(a) \[ \int \sqrt{1 + x^2} \, dx \quad \text{with} \quad u = 1 + x^2 \quad \frac{du}{dx} = 2x \quad \Rightarrow \quad \int u^{\frac{3}{2}} \, du = \frac{2}{3} u^{\frac{3}{2}} + C = \frac{2}{3} (1 + x^2)^{\frac{3}{2}} + C \]

(b) \[ \int \frac{\sin(\sqrt{x})}{\sqrt{x}} \, dx \quad \text{with} \quad u = \sqrt{x} \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}} \quad \Rightarrow \quad \int \sin(u) \, du = -\cos(u) + C = -\cos(\sqrt{x}) + C \]

(c) \[ \int \frac{2 + \ln(x)}{x} \, dx \quad \text{with} \quad u = 2 + \ln(x) \quad \frac{du}{dx} = \frac{1}{x} \quad \Rightarrow \quad \int u \, du = \frac{1}{2} u^2 + C = \frac{1}{2} (2 + \ln(x))^2 + C \]

(d) \[ \int \frac{2x}{1 + x^2} \, dx = \int \frac{2x}{1 + (x^2)^2} \, dx \quad \text{with} \quad u = x^2 \quad \frac{du}{dx} = 2x \quad \Rightarrow \quad \int \frac{1}{u^2} \, du = \arctan(u) + C = \arctan(x^2) + C \]

(e) \[ \int (1 - x^3)^{10} x^2 \, dx \quad \text{with} \quad u = 1 - x^3 \quad \frac{du}{dx} = -3x^2 \quad \Rightarrow \quad \int u^{10} (-\frac{1}{3}) \, du = -\frac{1}{3} u^{11} + C = -\frac{(1 - x^3)^{11}}{33} + C \]

**Example** Some integrals have some extra work in the substitution step, like below:

\[ \int p(p + 1)^5 dp \]

\[ u = p + 1 \]
\[ du = dp \]

Use the first equation to make a substitution

\[ = \int p(u^5) dp = \int (u^5) p dp \]

Use the second equation to make a substitution

\[ = \int (u^5) p du \]

Now return to the first equation to make a third substitution

\[ = \int (u^5)(u - 1) du \]

Distribute and integrate with the power rule

\[ = \int (u^6 - u^5) du = \left[ \frac{1}{7} u^7 \right] - \left[ \frac{1}{6} u^6 \right] + C \]

(2) \[ \int \frac{x}{\sqrt{x - 2}} \, dx \quad \text{with} \quad u = x - 2 \quad \frac{du}{dx} = 1 \quad \Rightarrow \quad \int \frac{1}{u^{\frac{1}{2}}} \, du = \int u^{-\frac{1}{2}} \, du = \frac{2}{3} u^{\frac{3}{2}} + C = \frac{2}{3} (x - 2)^{\frac{3}{2}} + C \]
(3) Calculate the following definite integrals.

(a) \[ \int_{0}^{\frac{\pi}{2}} \sin x \cos x \sqrt{1 - \sin^2 x} \, dx \]

\[ u = 1 - \sin^2 x \]
\[ du = -2 \sin x \cos x \, dx \]
\[ \frac{u}{2} = \sin^2 x \]
\[ \frac{1}{2} \int_{0}^{u} u \, du = \int_{0}^{u} u \, du \]
\[ u = 0 \]
\[ u = 1 \]
\[ \int_{0}^{1} \frac{u}{2} \, du = \int_{0}^{1} \frac{u}{2} \, du \]
\[ \frac{1}{2} \left[ \frac{u^2}{2} \right]_{0}^{1} = \frac{1}{4} \]

(b) \[ \int_{0}^{1} x^2 \sqrt{3 + x^2} \, dx \]

\[ u = 3 + x^2 \]
\[ du = 2x \, dx \]
\[ \frac{1}{2} \int_{0}^{4} u \, du = \int_{0}^{4} \frac{u}{2} \, du \]
\[ u = 3 + x^2 \]
\[ u = 0 \]
\[ \frac{1}{2} \int_{0}^{3} u \, du = \int_{0}^{3} \frac{u}{2} \, du \]

(4) A patient is experiencing blood loss at a rate of \( \frac{2}{1+t^2} \) pints per minute at time \( t \) (as \( t \) increases, the blood loss slows due to decreased pressure). Approximately how long before the patient loses 3 pints of blood after \( t = 0 \)?

\[ 3 = \int_{0}^{x} \frac{2}{1+t^2} \, dt \]
\[ \arctan(t) \bigg|_{0}^{x} = 3 \]
\[ \arctan(t) = \frac{3}{2} \]
\[ t = \tan(\frac{3}{2}) \approx 0 \text{ radians} \]

(5) Use an integral with \( y \) as the variable to find the area which is bounded by the \( y \)-axis and the curve \( x = \sin y \), between \( y = 0 \) and \( y = \pi \). Sketch the region, also.

\[ \int_{0}^{\pi} \sin(y) \, dy = -\cos(y) \bigg|_{0}^{\pi} = -1 - 1 = -2 \]

(6) Compute \( \frac{d}{ds} \int_{\cos(s)}^{s^{2}+2} \frac{1}{x^2 + 1} \, dx \).

\[ \frac{d}{ds} \int_{\cos(s)}^{s^{2}+2} \frac{1}{x^2 + 1} \, dx = \frac{d}{ds} \int_{\cos(s)}^{s^{2}+2} \frac{1}{x^2 + 1} \, dx \]

\[ = -\frac{1}{(\cos(s)^2 + 1)} \cdot (-\sin(s)) + \frac{1}{(s^{2}+2)^2 + 1} \]

"Time is a great teacher. Unfortunately, it gives way too many tests and it doesn't grade on a curve."