Fundamental Theorem of Calculus, Part I and II

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning. Suppose you know the following

\[
\int_{0}^{1} f(x) \, dx = 2 \quad \int_{0}^{2} f(x) \, dx = -1 \quad \int_{0}^{1} g(x) \, dx = 1 \quad \int_{0}^{2} g(x) \, dx = 0
\]

Use the properties of the integral to compute the following integrals.

1. \( \int_{0}^{1} 3f(x) + 2g(x) \, dx \)
2. \( \int_{0}^{1} f(x) \, dx \)
3. \( \int_{1}^{2} g(x) \, dx \)
4. \( \int_{0}^{2} 3 + 2f(x) - 5g(x) \, dx \)

5. Suppose that \( f \) is an odd function and \( g \) is an even function which are each integrable on the interval \([-3, 3]\). Given that \( \int_{0}^{3} f(x) \, dx = 4 \) and \( \int_{0}^{3} g(x) \, dx = 5 \), evaluate the following definite integral.

\( \int_{-3}^{3} (6f(x) + 8g(x)) \, dx \)

6. Use algebra and trig identities to evaluate the definite integral:

\[
\int_{\pi/6}^{\pi/3} \frac{\cos^3(x)\sin^2(x) + \cos^5(x)}{1 - \sin^2(x)} \, dx
\]
(7) Let \( g(x) = \int_{-19}^{x^2} (t - 169)^9 (5t - 245)^3 \, dt \). Determine the \( x \)-value for each local minimum on the graph of \( g(x) \).

Let \( f \) be the function whose graph is shown below and let \( g(x) = \int_0^x f(t) \, dt \).

(8) Find \( g(0) \), \( g(1) \), \( g(2) \), \( g(3) \), \( g(4) \), \( g(5) \), \( g(6) \).

(9) Estimate \( g(7) \).

(10) Where does \( g \) have an abs min and abs max on \([0.7]\)?

(11) Where does \( g \) have local maximum and minimum, if any?

(12) Where is \( g \) concave up and where is \( g \) concave down?