**Newton’s Method and Anti-derivatives**

**Instructions.** Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

(1) (a) If \( f \) is a differentiable function, what is the equation of the tangent line to the graph of \( f \) at some point \( x = a \)?

(b) Determine the \( x \)-intercept of the tangent line to the graph of \( f \) at the point \( x = a \).

(c) Explain why your solution to the previous problem looks a great deal like Newton’s formula for approximating roots.

(2) For which initial values in \([-1, 3]\) do you suspect Newton’s method will be successful in approximating the root of the function graphed. Explain why the method is likely to fail at the other values.

(3) If Newton’s Method is used to approximate a solution to the equation \( f(x) = 0 \), then it generates a sequence of approximations \( x_1, x_2, x_3, x_4, \ldots \). Which one of the following correctly shows how \( x_n \) can be used to determine the next approximation \( x_{n+1} \)?

\[
\begin{align*}
(a) \quad x_{n+1} &= \frac{x_n + f'(x_n)}{f(x_n)} \\
(b) \quad x_{n+1} &= x_n + \frac{f'(x_n)}{f(x_n)} \\
(c) \quad x_{n+1} &= \frac{x_n + f(x_n)}{f'(x_n)} \\
(d) \quad x_{n+1} &= x_n + \frac{f(x_n)}{f'(x_n)} \\
(e) \quad x_{n+1} &= \frac{x_n - f'(x_n)}{f(x_n)} \\
(f) \quad x_{n+1} &= x_n - \frac{f'(x_n)}{f(x_n)} \\
(g) \quad x_{n+1} &= \frac{x_n - f(x_n)}{f'(x_n)} \\
(h) \quad x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)}
\end{align*}
\]
(4) The tangent line to the graph \( y = f(x) \) at the point \( A(2, -1) \) is given by \( y = -1 + 4(x - 2) \). It is also known that \( f''(2) = 3 \).

(a) Assume that Newton’s Method is used to solve the equation \( f(x) = 0 \) and \( x_0 = 2 \) is the initial guess. Find the next approximation, \( x_1 \), to the solution.

(b) Assume that Newton’s Method is used to find a critical point for \( f \) and \( x_0 = 2 \) is the initial guess. Find the next approximation, \( x_1 \), to the critical point.

(5) If \( f''(x) = \sin x \) and \( f'(0) = 1 \) and \( f(0) = 2 \), determine \( f \).

(6) If \( f''(x) = \sqrt{x} \) and \( f'(1) = 2 \) and \( f(1) = 1 \) determine \( f \).

(7) If \( f(x) = \frac{9}{x} - 3e^{-0.4x} \), determine the general antiderivative function \( F(x) \).

(8) If \( f(x) = \frac{x^2 + 10}{x^2 + 1} \), determine the general antiderivative function \( F(x) \).

   Solution 1: Write the numerator as \( x^2 + 1 + 9 \)

   Solution 2: Perform long division