L’Hôpital’s Rule and Curve Sketching

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

1. Find the limit.

(a) \( \lim_{x \to 0} \frac{\sin 4x}{\tan 5x} \)

(b) \( \lim_{x \to 0} \frac{x}{1 - \cos x} \)

(c) \( \lim_{x \to 0} \frac{\cos x}{1 - \sin x} \)

(d) \( \lim_{x \to \infty} \frac{x^n}{e^x} \) (any \( n \))

(e) \( \lim_{x \to 0^+} \left( \frac{1}{x} - \frac{1}{e^x - 1} \right) \)

(f) \( \lim_{x \to 1^+} (\ln(x^7 - 1) - \ln(x^5 - 1)) \)
2. Consider the function $f(x) = \frac{x^2}{e^x}$.

(a) Where is $f$ equal to 0 or undefined? Where is $f$ positive, negative?

(b) What are the horizontal, vertical asymptotes of $f$? How does $f$ behave as $x \to \pm\infty$?

(c) Where are the critical points of $f$? Where is $f$ increasing, decreasing?

(d) Where is $f'' = 0$? Where is $f$ concave up, down?

(e) Sketch the graph of $f(x)$. Be sure your graph clearly indicates the information you determined earlier.