MORE RELATED RATES

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

(1) Two carts, A and B, are connected by a rope 50 ft long that passes over a pulley P. The point Q is on the floor $h = 10$ ft directly beneath $P$ and between the carts. Cart A is being pulled away from Q at a speed of 3 ft/s. How fast is cart B moving toward Q at the instant when cart A is 5 ft from Q?

From the diagram:

\[ x, y \rightarrow \text{distances of carts from point Q} \]
\[ a, b \rightarrow \text{lengths of ropes} \]

**Know:**
\[ \frac{dx}{dt} = 3 \text{ ft/sec} \]
\[ a + b = 50 \]

**Want:**
\[ \frac{dy}{dt} \text{ when } x = 5 \]

\[ a + b = 50 \]
\[ \sqrt{x^2 + 100} + \sqrt{y^2 + 100} = 50 \]

**Eqns:**
\[ x^2 + 100 = a^2 \]
\[ y^2 + 100 = b^2 \]
\[ a = \sqrt{x^2 + 100} \]
\[ b = \sqrt{y^2 + 100} \]

**Differentiate:**
\[ \frac{d}{dt}\left(\sqrt{x^2 + 100}\right) + \frac{d}{dt}\left(\sqrt{y^2 + 100}\right) = 50 \]

\[ \frac{1}{2}\left(x + 100\right)^{-1/2} \cdot \frac{dx}{dt} + \frac{1}{2}\left(y + 100\right)^{-1/2} \cdot \frac{dy}{dt} = 0 \]

\[ \frac{x}{\sqrt{x^2 + 100}} \cdot \frac{dx}{dt} + \frac{y}{\sqrt{y^2 + 100}} \cdot \frac{dy}{dt} = 0 \]

\[ \frac{dy}{dt} = -\frac{x}{\sqrt{y^2 + 100}} \cdot \frac{\sqrt{y^2 + 100}}{y} \cdot \frac{dx}{dt} \]

Plug in $\frac{dx}{dt} = 3$

\[ x = 5 \]

**When x = 5:**
\[ \frac{dy}{dt} = \frac{-5}{\sqrt{5^2 + 100}} \cdot \frac{\sqrt{5^2 + 100}}{5} \cdot \frac{3}{37.5} \approx -1.4 \text{ ft/sec} \]

B is moving toward Q at 1.4 ft/sec.
(2) A runner sprints around a circular track of radius 100 m at a constant speed of 5 m/s. The runner's friend is standing at a distance 200 m from the center of the track. How fast is the distance between the friends changing when the distance between them is 150 m? Start by drawing a picture! Hint: you will need to translate the speed of 5 m/s into a rate of change for the angle between the runner, the center of the circle, and the runner's friend; the rate of change should be measured in radians/s.

\[ \text{want: } \frac{dx}{dt} \text{ when } x = 150 \]

\[ \text{Know Runner is moving around circle at 5 m/s} \]

\[ L = \frac{\theta}{2\pi} \cdot 2\pi (100) \]

\[ L = 100 \theta \]

\[ \frac{dl}{dt} = 5 \text{ m/s} \]

\[ \frac{dl}{dt} = 100 \frac{d\theta}{dt} \]

\[ 0.5 \text{ rad/sec} = \frac{5}{100} = \frac{d\theta}{dt} \]

\[ x = 100^2 + 200^2 - 2(100)(200) \cos \theta \]

\[ 2x \frac{dx}{dt} = 40000 \sin \theta \frac{d\theta}{dt} \]

\[ \frac{dx}{dt} = \frac{20000 \sin \theta}{x} \frac{d\theta}{dt} \]

\[ \text{Plug in } \frac{d\theta}{dt} = 0.05 \]

\[ x = 150 \]

\[ \sin \theta = 0.73 \]

\[ \frac{dx}{dt} \approx 4.9 \text{ m/s} \]

\[ (150)^2 = 50000 - 40000 \cos \theta \]

\[ \cos \theta = 0.6875 \]

\[ \sin \theta = \sqrt{1 - (0.6875)^2} = 0.73 \]