Related Rates Word Problems

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

1) Your extra thick maple syrup tends to spread out in a circle about 1/8 inch thick. If you pour out the syrup at a rate of 1.8 cubic inches per second (that's just under 2 tablespoons a second) how fast is the radius of the pool of syrup increasing when the radius is 1 inch? What about 5 inches (that is probably too much syrup)?

(a) What rate is known?

\[
\frac{dV}{dt} = 1.8 \text{ in}^3/\text{sec}
\]

(b) What rate do you want to find?

\[
\frac{dr}{dt} \quad \text{when} \quad r = 1 \text{ in}
\]

(c) Write an equation.

\[
V = \pi r^2 h \quad V = \frac{\pi}{8} r^2 \quad \text{(Thickness, or height, is constantly)} \quad \frac{1}{8} \text{ in}
\]

(d) Take \(d/dt\) of both sides.

\[
\frac{dV}{dt} = \frac{\pi}{4} r \frac{dr}{dt}
\]

(e) Plug in known values.

\[
1.8 = \frac{\pi}{4} (1) \frac{dr}{dt} \quad 1.8 = \frac{\pi}{4} (5) \frac{dr}{dt}
\]

(f) Find the answer to the question. Include units in your answer.

\[
\frac{dr}{dt} = \frac{4}{\pi} (1.8) \text{ in/s} \approx 2.3 \text{ in/sec} \quad \text{when} \quad r = 1 \text{ in}
\]

\[
\frac{dr}{dt} = \frac{4}{5 \pi} (1.8) \text{ in/s} \approx 0.46 \text{ in/sec} \quad \text{when} \quad r = 5 \text{ in}
\]
(2) One car leaves a given point and travels north at 30 mph. Another car leaves 1 HOUR LATER, and travels west at 40 mph. At what rate is the distance between the cars changing at the instant the second car has been traveling for 1 hour?

\[
\frac{dy}{dt} = 30 \text{ mph} \quad \frac{dx}{dt} = 40 \text{ mph}
\]

\[x^2 + y^2 = z^2\]

\[
2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}
\]

\[
\frac{dz}{dt} = \frac{2 \frac{dx}{dt} + 2y \frac{dy}{dt}}{2x}
\]

\[
\frac{dz}{dt} \approx 47,150 \text{ mph}
\]

(3) A 50 ft ladder is placed against a large building. The base of the ladder is resting on an oil spill, and it slips at the rate of 3 ft. per minute. Find the rate of change of the height of the top of the ladder above the ground at the instant when the base of the ladder is 30 ft. from the base of the building.

\[
\frac{dx}{dt} = 3 \quad x^2 + h^2 = 50^2
\]

\[
2x \frac{dx}{dt} + 2h \frac{dh}{dt} = 0
\]

\[
\frac{dh}{dt} = -\frac{x \frac{dx}{dt}}{h}
\]

\[
\frac{dh}{dt} = -2.25 \text{ ft/min}
\]

(4) The radius of a cylinder is increasing at a rate of 1 meter per hour, and the height of the cylinder is decreasing at a rate of 4 meters per hour. At a certain instant, the base radius is 5 meters and the height is 8 meters. What is the rate of change of the volume of the cylinder at the instant?

\[
V = \pi r^2h
\]

\[
\frac{dV}{dt} = (\pi r^2) \frac{dr}{dt} + (2\pi r \frac{dr}{dt}) \frac{dh}{dt}
\]

\[
= \pi (5)^2(-4) + 2\pi (5)(1)(8) = -120 \pi \text{ m}^3/\text{hour}
\]

(5) A person who is 6 feet tall is walking away from a lamp post at the rate of 40 feet per minute. When the person is 10 feet from the lamp post, his shadow is 20 feet long. Find the rate at which the length of the shadow is increasing when he is 30 feet from the lamp post.

\[
\frac{dx}{dt} = 40 \quad \frac{ds}{dt} = 80 \text{ ft/min}
\]

\[
\frac{x + s}{a} = \frac{s}{b}
\]

\[
\frac{2a}{b} \cdot \frac{dx}{dt} + \frac{a}{b} \frac{ds}{dt} = \frac{ds}{dt}
\]

\[
h = 180 \quad \text{in} \quad \frac{a}{b} \frac{dx}{dt} = \frac{ds}{dt}
\]

\[
h = q
\]
2. One car leaves a given point and travels north at 30 mph. Another car leaves 1 HOUR LATER, and travels west at 40 mph. At what rate is the distance between the cars changing at the instant the second car has been traveling for 1 hour?

3. A 50ft ladder is placed against a large building. The base of the ladder is resting on an oil spill, and it slips at the rate of 3 ft. per minute. Find the rate of change of the height of the top of the ladder above the ground at the instant when the base of the ladder is 30 ft. from the base of the building.

4. A water tank has the shape of an inverted circular cone with base radius 2 m and height 4m. If water is being pumped into the tank at a rate of 2 m³/min, find the rate at which the water level is rising when the water is 3 m deep.

5. A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.