Math 234 Spring 2014 Exam 1 Version 1
Monday, February 17, 2014

Name: __________________________ UIN: ____________________

Circle the section you are registered for:

ADA (9:00 – Neriman)  BDA (9:00 – Dileep)  CDA (8:00 – Nima)
ADB (10:00 – Neriman)  BDB (10:00 – Dileep)  CDB (9:00 – Nima)
ADC (11:00 – Itziar)  BDC (11:00 – Jiashun)  CDC (11:00 – Michael)
ADD (12:00 – Itziar)  BDD (12:00 – Sean)  CDD (12:00 – Michael)
ADE (1:00 – Malik)  BDE (1:00 – Argen)  CDE (1:00 – Jaehoon)
ADF (2:00 – Malik)  BDF (2:00 – Argen)  CDF (2:00 – Claire)
ADG (3:00 – Alessandro)  BDG (3:00 – Albert)  CDG (3:00 – Claire)
ADH (4:00 – Alessandro)  BDH (4:00 – Albert)  CDH (11:00 – Sean)

(1) You should have already checked into the exam and had your calculator approved. No exam will be accepted from a student who does not check in before they start the exam.

(2) No baseball caps, hoodies, etc. or dark sunglasses. All hats are to be removed.

(3) All book bags should be closed and placed at the front of the classroom. Do not reach into your bag for anything during the exam. If you need extra pencils, pull them out NOW.

(4) No cell phones. Turn them off now. If you are seen with a cell phone in hand during the exam it will be considered cheating and you will be asked to leave. This includes using it as a time-piece.

(5) No iPods or MP3s players, etc. Same rules as with cell phones.

(6) If you have a question, raise your hand and a proctor will come to you. If you have to use the bathroom, do so NOW. You will not be permitted to leave the room and return during the exam.

(7) Every exam is worth a total of 100 points. Check to see that you have all of the pages. Including this cover sheet, each exam has 7 sides. Each problem is worth 16 points. You will receive 4 points for your name and section.

(8) Be sure to print your proper name clearly at the top of this page. Also circle the section for which you are registered.

(9) If you finish early, quietly and respectfully get up and hand in your exam.

(10) When time is up, you will be instructed to put down your writing utensil, close your exam and remain seated. Anyone seen continuing to write after time is called will have their exam marked and lose all points on the page they are writing on.

(11) To ensure that you receive full credit, show all of your work.

(12) Good luck. You have 60 minutes to complete this exam.
(1) Compute the following limits or state that they do not exist.

(a) \[
\lim_{x \to -2} \frac{x^2 + x - 2}{x^2 - x - 6}
\]
\[
= \lim_{x \to -2} \frac{(x - 1)(x + 2)}{(x - 3)(x + 2)}
\]
\[
= \lim_{x \to -2} \frac{x - 1}{x - 3}
\]
\[
= \frac{-2 - 1}{-2 - 3}
\]
\[
= \frac{3}{5}
\]

(b) \[
\lim_{x \to \infty} \frac{3x^3 - 4x^2 + 2}{(2x + 1)^3}
\]
\[
= \lim_{x \to \infty} \frac{3x^3 - 4x^2 + 2}{8x^3 + 12x^2 + 6x + 1}
\]
\[
= \lim_{x \to \infty} \frac{3x^3 - 4x^2 + 2}{8x^3 + 12x^2 + 6x + 1} \cdot \frac{1}{x^3}
\]
\[
= \lim_{x \to \infty} \frac{3 - \frac{4}{x} + \frac{2}{x^3}}{8 + \frac{12}{x} + \frac{6}{x^2} + \frac{1}{x^3}}
\]
\[
= \frac{3 - 0 + 0}{8 + 0 + 0 + 0}
\]
\[
= \frac{3}{8}
\]
Charlotte’s Sweet Shoppe specializes in producing boxes of chocolate covered toffee. Each box costs $4 to produce and can be sold for $10. Assume that Charlotte’s has fixed daily overhead costs of $150.

(a) Write a function for Charlotte’s total daily cost if she produces \( x \) boxes of chocolate covered toffee.

(b) Assuming that Charlotte sells every box of chocolate covered toffees that she produces, what is the daily revenue function?

(c) How many boxes of toffee will she need to sell each day in order to break even?

(a) \( \text{Cost} = 150 + 4x \)

(b) \( \text{Revenue} = 10x \)

(c) The break even point is the point at which revenue is equal to cost. We have

\[
\text{Cost} = \text{Revenue} \\
150 + 4x = 10x \\
150 = 6x \\
25 = x
\]

So she needs to produce 25 boxes of chocolate covered toffee each day in order to break even.
(3) (a) Which of the following is the correct definition of the derivative of $f(x)$?

(i) $f'(x) = \lim_{h \to 0} \frac{f(x + h) + f(x)}{h}$

(ii) $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$

(iii) $f'(x) = \frac{f(x + h) - f(x)}{h}$

(iv) $f'(x) = \lim_{x \to 0} \frac{f(x + h) - f(x)}{x}$

(b) Use the definition of the derivative to compute $f'(x)$ where $f(x) = x^2 + 3x$

\[
\begin{align*}
    f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\
    &= \lim_{h \to 0} \frac{[(x + h)^2 + 3(x + h)] - [x^2 + 3x]}{h} \\
    &= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h} \\
    &= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h} \\
    &= \lim_{h \to 0} \frac{2xh}{h} + \lim_{h \to 0} \frac{h^2 + 3h}{h} \\
    &= \lim_{h \to 0} 2x + \lim_{h \to 0} \left( \frac{h^2}{h} + 3 \right) \\
    &= 2x + 3
\end{align*}
\]
(4) A concert promoter is trying to decide what price to charge per ticket. She finds that she can sell 1000 tickets at $50 each. For every $1 increase in ticket price, she will sell 10 fewer tickets.

Let $x$ be the number of $1 price increases.

(a) Express $p$, the price of each ticket, as a function of $x$.

$$ p = 50 + x $$

(b) Find an expression for $n$, the number of tickets sold, after $x$ price increases

$$ n = 1000 - 10x $$

(c) Find an expression for the total revenue after $x$ price increases.

$$ \text{Revenue} = (\text{tickets sold}) \cdot (\text{price per ticket}) $$

$$ R = (1000 - 10x) \cdot (50 + x) $$

$$ = 50,000 + 500x - 10x^2 $$

(d) What value of $x$ maximizes revenue?

The revenue will be maximized at the vertex of the parabola.

$$ \frac{-b}{2a} = \frac{-500}{2 \cdot -10} = \frac{500}{20} = 25 $$

So the revenue is maximized at $x = 25$

(e) What price should she charge for each ticket to maximize revenue? How many tickets will she sell at that price?

When $x = 25$, $p = 50 + 25 = 75$ and $n = 1000 - 25 \cdot 10 = 750$.

(f) What will her maximum revenue be?

When $x = 25$, we have

$$ R(25) = (1000 - 10 \cdot 25)(50 + 25) = 750 \cdot 75 = 56250 $$
(5) Find all values $k$ so that $f(x)$ is continuous for all real values of $x$.

$$f(x) = \begin{cases} x^2 + 2x & \text{if } x \leq 2 \\ x^3 + kx^2 + 4 & \text{if } x > 2 \end{cases}$$

Since both pieces of our piecewise defined function are polynomials, the only possible point of discontinuity occurs when we transfer from the first function definition to the second. So long as our one sided limits at this point align, we will have continuity. Thus, we compute

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\lim_{x \to 2^-} x^2 + 2x = \lim_{x \to 2^+} x^3 + kx^2 + 4$$

$$2^2 + 2 \cdot 2 = 2^3 + k \cdot 2^2 + 4$$

$$8 = 12 + 4k$$

$$-4 = 4k$$

$$-1 = k$$

So our function is continuous everywhere when $k = -1$. 
(6) How long will it take $1 to triple at an annual inflation rate of 8% compounded continuously?

Our formula for continuous compounding is given by $A(t) = Pe^{rt}$. Here we know that $P = 1$, $r = 0.08$, and $A(t) = 3$. To find $t$, we take

$$A = Pe^{rt}$$
$$3 = 1e^{0.08t}$$
$$\ln 3 = \ln(e^{0.08t})$$
$$\ln 3 = 0.08t$$
$$t = \frac{\ln 3}{0.08}$$
$$t \approx 13.73$$

So the money will take about 13.73 years to triple in value.
(7) Find the domain of the following functions:

(a) \( y = \frac{\sqrt{x - 2}}{2x + 3} \)

Observe that the numerator is undefined whenever \( x - 2 < 0 \) or, alternately whenever, \( x < 2 \). Further, the denominator cannot be zero, so

\[
2x + 3 \neq 0 \implies 2x \neq -3 \implies x \neq \frac{-3}{2}
\]

But \(-3/2 < 2\), so it provides no new information. Hence, the domain of the function is

\( \{ x : x \geq 2 \} \)

or alternately

\( [2, \infty) \)

(b) \( y = ln(x + 7) \)

Recall that logarithmic functions are defined for all positive values. So we need

\( x + 7 > 0 \implies x > -7 \)

Hence, the domain of the function is

\( \{ x : x > -7 \} \)

or alternately

\( (-7, \infty) \)
(8) Evaluate the following:

(a) \(\log_3(81)\)

We have
\[
\log_3(81) = x \implies 3^x = 81 \implies x = 4
\]

(b) \(\left(\frac{1}{3}\right)^{-3}\)

We have
\[
\left(\frac{1}{3}\right)^{-3} = \left[\left(\frac{1}{3}\right)^{-1}\right]^3 = 3^3 = 27
\]

(c) \(\log_4(8)\)

We have
\[
\log_4(8) = x \implies 4^x = 8
\]

Since
\[
8 = 4 \cdot 2 = 4 \cdot \sqrt{4} = 4^{3/2}
\]

it follows that
\[
\log_4(8) = \frac{3}{2}
\]

(d) \(4^{-3/2}\)

We have
\[
4^{-3/2} = (4^{3/2})^{-1} = (4 \cdot \sqrt{4})^{-1} = 8^{-1} = \frac{1}{8}
\]