(1) Use the properties of exponents and logarithms to simplify
(a) \((-2t^{-3})(3t^{2/3})\)
(b) \(\ln(x^2\sqrt{4-x^2})\)

(a) 
\[(-2t^{-3})(3t^{2/3}) = -2 \cdot 3 \cdot t^{-3} \cdot t^{2/3} = -6t^{-3+2/3} = -6t^{-7/3}\]

(b) 
\[\ln(x^2\sqrt{4-x^2}) = \ln x^2 + \ln(4-x^2)^{1/2}\]
\[= 2\ln x + \frac{1}{2}\ln[(2-x)(2+x)]\]
\[= 2\ln x + \frac{1}{2}[\ln(2-x) + \ln(2+x)]\]

(2) Differentiate
\[g(x) = x^2 \ln x\]

We need to use the product rule here. We have
\[g'(x) = 2x \ln x + x^2 \cdot \frac{1}{x}\]
\[= 2x \ln x + x\]

(3) Compute \(\int (3t^2 - \sqrt{5}t + 2)\,dt\)

\[\int (3t^2 - \sqrt{5}t + 2)\,dt = 3\frac{t^{2+1}}{2+1} - \sqrt{5}\frac{t^{1+1}}{1+1} + 2t + C\]
\[= t^3 - \frac{\sqrt{5}}{2}t^2 + 2t + C\]
(4) Evaluate \( \int \frac{1}{x \ln x} \, dx \).

We need to use substitution for this problem. Take

\[ u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x} \, dx \]

This substitution gives us

\[
\int \frac{1}{x \ln x} \, dx = \int \frac{1}{\ln x} \cdot \frac{1}{x} \, dx = \int \frac{1}{u} \, du = \ln |u| + C = \ln |\ln x| + C
\]

(5) Suppose $1000 is invested at an annual interest rate of 5%. Compute the balance after 10 years of the interest is compounded

(a) Quarterly
(b) Monthly
(c) Continuously

Recall first that the formula for interest compounded \( n \) times per year is

\[ P(t) = P_0 (1 + \frac{r}{n})^{nt} \]

and the formula for interest compounded continuously is given by

\[ P(t) = P_0 e^{rt} \]

In all of these cases, we have \( P_0 = 1000, r = .05, \) and \( t = 10. \)

(a) We compound 4 times per year, so \( n = 4 \) and the value of our investment after 10 years is

\[
P(10) = 1000 (1 + \frac{.05}{4})^{4 \cdot 10} = 1000(1.0125)^{40} \text{ (note that this would be acceptable for the answer on the midterm)}
\]

(b) We compound 12 times per year, so \( n = 4 \) and the value of our investment after 10 years is

\[
P(10) = 1000 (1 + \frac{.05}{12})^{12 \cdot 10} = 1000(1.00417)^{120}
\]

(c) Since we are compounding continuously, we have

\[ P(10) = 1000 e^{.05 \cdot 10} = 1000 e^{.5} \]
(6) Use logarithmic differentiation to find the derivative of

\[ f(x) = x^2e^{-x}(3x + 5)^3 \]

Taking the natural logarithm of each side and simplifying, we have

\[ \ln(f(x)) = \ln(x^2e^{-x}(3x + 5)^3) \]
\[ = \ln x^2 + \ln e^{-x} + \ln(3x + 5)^3 \]
\[ = 2\ln x - x + 3\ln(3x + 5) \]

Now, differentiating each side, we have

\[ \frac{f'(x)}{f(x)} = 2\frac{1}{x} - 1 + 3 \frac{3}{3x + 5} \]
\[ = \frac{2}{x} - 1 + \frac{9}{3x + 5} \]

Multiplying both sides by \( f(x) \), we arrive at our final answer of

\[ f'(x) = \left[ \frac{2}{x} - 1 + \frac{9}{3x + 5} \right] \cdot [x^2e^{-x}(3x + 5)^3] \]

(7) Evaluate \( \ln \frac{e^3 \sqrt{e}}{e^{1/3}} \) using properties of the natural logarithm.

We have

\[ \ln \frac{e^3 \sqrt{e}}{e^{1/3}} = \ln(e^3 \sqrt{e}) - \ln(e^{1/3}) \]
\[ = \ln e^3 + \ln e^{1/2} - \ln e^{1/3} \]
\[ = 3 \frac{1}{2} - \frac{1}{3} \]
\[ = 3 \frac{3}{6} + \frac{2}{6} = \frac{19}{6} \]
(8) Evaluate $\int_0^1 e^{-x}(4 - e^x)\,dx$ using the fundamental theorem of calculus.

\[
\int_0^1 e^{-x}(4 - e^x)\,dx = \int_0^1 4e^{-x} - e^{-x}e^x\,dx = \int_0^1 4e^{-x} - 1 \,dx = 4\frac{e^{-1}}{-1} - t\bigg|_0^1 = -4e^{-x} - t\bigg|_0^1 = [-4e^{-1} - 1] - [-4e^0 - 0] = -\frac{4}{e} - 1 + 4 = 3 - \frac{4}{e}
\]

(9) A book publisher estimates that his profits from the sale of a particular book will be

\[P(x) = 1000e^{\sqrt{3}x^{1/2}}\]

when $x$ thousand copies of the book are produced. The publisher is currently planning to produce 3000 copies of the book. Use calculus to estimate the marginal increase in profit when an additional 1000 copies of the book are produced.

In order to find marginal profit, we need to compute the derivative of the profit function and evaluate it at our “starting point”. We have

\[
P(x) = 1000e^{\sqrt{3}x^{1/2}} \Rightarrow P'(x) = 1000e^{\sqrt{3}x^{1/2}} \cdot \sqrt{3} \cdot \frac{1}{2}x^{-1/2} = 500\sqrt{3} \cdot \frac{1}{\sqrt{x}} e^{\sqrt{3}x^{1/2}}
\]

Since we start with 3000 copies and $x$ is given in 1000s, we have $x = 3$. Evaluating $P'(3)$ we have

\[
P'(3) = 500\sqrt{3} \cdot \frac{1}{\sqrt{3}} e^{(\sqrt{3})(3^{1/2})} = 500e^3
\]
(10) Find the function \( f(x) \) which has derivative \( f'(x) = 3x^2 + 6x - 2 \) and passes through the point \((0, 6)\).

We need to begin by finding the integral of \( f'(x) \). We have

\[
 f(x) = \int 3x^2 + 6x - 2 \, dx = x^3 + 3x^2 - 2x + C  
\]

\[
 f(0) = 6 = 0^3 + 3 \cdot 0^2 - 2 \cdot 0 + C 
\]

\[
 6 = C 
\]

\[
 f(x) = x^3 + 3x^2 - 2x + 6 
\]