(1) Compute the derivative of \( f(x) = x^2e^x + 1 \).

We need to use the product rule. We have

\[
f'(x) = 2xe^x + x^2e^x + 0 = xe^x(2 + x)
\]

(2) Use logarithmic differentiation to compute the derivative of \( h(x) = \frac{(x + 1)^7e^{3x}}{(2x - 4)^3} \).

We begin by taking the natural logarithm of both sides. We have

\[
\ln[h(x)] = \ln[(x + 1)^7e^{3x}] - \ln[(2x - 4)^3]
\]

\[
= 7 \ln(x + 1) + 3x - 3 \ln(2x - 4)
\]

Differentiating then gives us

\[
\frac{h'(x)}{h(x)} = \frac{7}{x + 1} + 3 - \frac{6}{2x - 4}
\]

\[
h'(x) = \left[ \frac{7}{x + 1} + 3 - \frac{6}{2x - 4} \right] h(x)
\]

(3) Compute the integral

\[
\int \frac{1}{x^2} \, dx
\]

We have

\[
\int \frac{1}{x^2} \, dx = \int x^{-2} \, dx = \frac{1}{-2 + 1} x^{-2+1} + C = -x^{-1} + C = \frac{-1}{x} + C
\]
(4) Compute the integral

$$\int (3x^2 + 1)e^{x^3 + x} \, dx$$

We need to use substitution here in order to find the antiderivative. Take

$$u = x^3 + x$$

Then

$$\frac{du}{dx} = 3x^2 + 1 \Rightarrow du = 3x^2 + 1 \, dx$$

Substituting, we have

$$\int (3x^2 + 1)e^{x^3 + x} \, dx = \int e^{x^3 + x} \cdot (3x^2 + 1) \, dx$$

$$= \int e^u \, du$$

$$= e^u + C$$

$$= e^{x^3 + x} + C$$

(5) Find all points where the tangent to the graph of $g(x) = x \ln x$ is horizontal.

To find the points where the tangent is horizontal, we need to take the derivative of $g(x)$ and find where it is equal to zero. We have

$$g'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x}$$

$$= \ln x + 1$$

$$0 = \ln x + 1$$

$$-1 = \ln x$$

$$e^{-1} = e^{\ln x}$$

$$\frac{1}{e} = x$$

So the tangent line is horizontal at the point $(e^{-1}, g(e^{-1})) = (e^{-1}, -e^{-1})$
A manufacturer estimates that the marginal cost of producing $q$ units of a certain commodity is $C'(q) = 3q^2 - 24q + 48$ dollars per unit. If the cost of producing 10 units is $5000, what is the cost of producing 30 units?

Observe that we are given $C'(q)$ and we wish to find $C(q)$, its antiderivative. We have

$$C(q) = \int (3q^2 - 24q + 48)\,dq$$
$$= q^3 - 12q^2 + 48q + K$$

$5000 = C(10) = (10)^3 - 12(10)^2 + 48(10) + K$

$5000 = 1000 - 1200 + 480 + K$

$5000 = 280 + K$

$4720 = K$

So we have

$$C(q) = q^3 - 12q^2 + 48q + 4720$$

$C(30) = (30)^3 - 12(30)^2 + 48(30) + 4720$

$= 27000 - 12(900) + 48(30) + 4720$

$= 22360$

So it will cost them $22,360 to produce 30 units.
(7) The population of a certain bacterial colony \( t \) hours after antibiotic treatment is begun is given by the formula

\[ P(t) = \frac{100e^{0.03t}}{t + 1} \]

(a) What will the population be 5 hours after treatment is begun? 10 hours? 20 hours?
(b) Will the population of the colony ever be reduced to zero?

(a) We have

\[
\begin{align*}
P(5) &= \frac{100e^{0.03\cdot5}}{5 + 1} \\
&= \frac{100e^{0.15}}{6} \\
&= 19.36 \\
P(10) &= \frac{100e^{0.03\cdot10}}{10 + 1} \\
&= \frac{100e^{0.3}}{11} \\
&= 12.27 \\
P(20) &= \frac{100e^{0.03\cdot20}}{20 + 1} \\
&= \frac{100e^{0.6}}{21} \\
&= 8.68
\end{align*}
\]

(b) To see if the population will ever be reduced to zero we take \( P(t) = 0 \) and try to solve for \( t \). We have

\[
0 = \frac{100e^{0.03t}}{t + 1} \\
0 = 100e^{0.03t} \\
0 = e^{0.03t}
\]

But we know that \( e^t > 0 \) for all \( t \), and so this has no solution. Hence the bacterial population will never be reduced to zero.
A study indicates that $t$ months from now the population of a certain town will be growing at the rate of $P'(t) = 5 + 3t^{2/3}$ people per month. Use a definite integral to compute how much the population of the town will rise over the next 8 months.

We need to compute

$$
\int_0^8 5 + 3t^{2/3} \, dt = 5t + 3 \cdot \frac{1}{2/3 + 1} t^{2/3+1} \bigg|_0^8 \\
= 5t + \frac{9}{5} t^{5/3} \bigg|_0^8 \\
= \left[ 5 \cdot 8 + \frac{9}{5} (8)^{5/3} \right] - \left[ 5 \cdot 0 + \frac{9}{5} (0)^{5/3} \right] \\
= 40 + \frac{9}{5} \cdot 32 - [0 + 0] \\
= 40 + \frac{288}{5} \\
= \frac{488}{5} \approx 98
$$

So the population will increase by about 98 people in the next 8 months.

(9) Simplify the following:

(a) $(25x^6)^{3/2}$

(b) $\log_3 27(2x + 5)^7$

(a) We have

$$(25x^6)^{3/2} = 25^{3/2} \cdot (x^6)^{3/2} \\
= (\sqrt[3]{25})^3 \cdot (\sqrt[3]{x^6})^3 \\
= 5^3 \cdot (x^3)^3 \\
= 125x^9$$

(b) We have

$$\log_3 27(2x + 5)^7 = \log_3 27 + \log_3 (2x + 5)^7 \\
= \log_3 3^3 + 7 \log_3 (2x + 5) \\
= 3 + 7 \log_3 (2x + 5)$$
(10) Compute the integral

\[ \int y^3 \left( 2y + \frac{1}{y} \right) dy \]

We have

\[ \int y^3 \left( 2y + \frac{1}{y} \right) dy = \int 2y^4 + y^2 dy \]

\[ = 2 \left( \frac{1}{4+1}y^{4+1} + \frac{1}{2+1}y^{2+1} + C \right) \]

\[ = 2 \left( \frac{1}{5}y^5 + \frac{1}{3}y^3 + C \right) \]