MATH 221 Calculus I
Exam 2 Review Multiple Choice Questions

December 5th, 2019

Solutions
Determine the limit.

\[
\lim_{x \to 0^-} \frac{\cos(2x + \pi)}{1 - e^x}
\]

(A) \(\infty\)

(B) \(-\infty\)

(C) 0

(D) The limit does not exist
\[
\lim_{x \to 0^-} \frac{\cos(2x + \pi)}{1 - e^x} = \frac{\cos(\pi)}{0} = \frac{-1}{0^+} = [-\infty]
\]

Or, we could ask
\[
\lim_{x \to 0^-} \frac{\cos(2x + \pi)}{1 - e^{-x}} = \frac{\cos(\pi)}{0} = \frac{-1}{0^-} = [\infty]
\]
A street light is mounted on top of a 20 ft tall pole. A woman 5 ft tall walks directly toward the pole with a speed of 7.5 ft/s. How quickly is the length of her shadow decreasing when she is 13 ft from the pole?

(A) 2 ft/sec

(B) 2.25 ft/sec

(C) 2.5 ft/sec

(D) The correct answer is not here
Given
\[
\frac{dx}{dt} = -7.5 \text{ ft/sec}
\]

Want
\[
\frac{dy}{dt} \text{ when } x = 13
\]

\[
\frac{20}{x+y} = \frac{5}{y}
\]

\[
d0y = 5x + 5y
\]

\[
15y = 5x
\]

\[
y = \frac{1}{3} x
\]

\[
\frac{d}{dt} \left( y \right) = \frac{d}{dt} \left( \frac{1}{3} x \right)
\]

\[
\frac{dy}{dt} = \frac{1}{3} \frac{dx}{dt}
\]

\[
= \frac{1}{3} (-7.5) = -2.5
\]

\[
= -\frac{15}{6} = -\frac{5}{2}
\]

\[
7.5 = 15 \frac{1}{2} = 15\frac{1}{2}
\]
A street light is mounted at the top of a 630 cm pole. As a woman walks away from the pole, the tip of her shadow is moving 40% faster than she is moving. What is the woman's height?

(A) 180 cm
(B) 200 cm
(C) 220 cm
(D) The correct answer is not here
\[
\frac{630}{y} = \frac{h}{y-x}
\]

\[
given \quad \frac{dy}{dt} = \frac{dx}{dt} + 0.4 \frac{dx}{dt} = 1.4 \frac{dx}{dt}
\]

\[
want \quad h
\]

\[
630 (y-x) = hy
\]

\[
d\frac{d}{dt} (630y - 630x) = \frac{d}{dt}(hy)
\]

\[
630 \frac{dy}{dt} - 630 \frac{dx}{dt} = h \frac{dy}{dt}
\]

\[
h = 630 \frac{dy}{dt} - 630 \frac{dx}{dt}
\]

\[
h = 1.4(\frac{dx}{dt}) - 630 \frac{dx}{dt}
\]

\[
h = 630 \frac{dx}{dt} \left[ 1.4 - 1 \right] = 630 \frac{(4/10)}{(14/10)} = 180
\]
Section 3.9: Types of Related Rates

Word Problems

- Light Pole / Shadow

- Angle of Elevation

\[ \sin \theta, \cos \theta, \tan \theta \]

- Pythagorean Theorem

- Similar Triangles

- Ladders Sliding

Trig or Pythagorean
Find the x-coordinate of each inflection point on the graph of 
\[ f(x) = 3x^5 - 50x^4 + 250x^3 - 314x + 42 \]

\[ f'(x) = 15x^4 - 200x^3 + 750x^2 - 314 \]
\[ f''(x) = 60x^3 - 600x^2 + 1500x \]

(A) \( x=5 \)

(B) \( x=0 \)

(C) \( x=0, x=5 \)

(D) The correct answer is not here
Find the absolute maximum value of the function 
\( f(x) = 3x^4 - 16x^3 + 18x^2 \) on the interval \([-1, 4]\)

\[ f'(x) = 12x^3 - 48x^2 + 36x \]

\( f'(x) = 0 \) (or undefined)

\[ 12x(x^2 - 4x + 3) = 0 \]
\[ 12x(x-3)(x-1) = 0 \]
\[ x = 0, 3, 1 \]

<table>
<thead>
<tr>
<th>X</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-27</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>-1</td>
<td>37</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
</tbody>
</table>

(A) 37
(B) -1
(C) 5
(D) The correct answer is not here
Express $10 \ln(2) - 3 \ln(10)$ as a single logarithm. Now use a linear approximation to estimate its value. Simplify and write your answer in decimal form.

(A) 0.024

(B) 0.023

(C) 0.022

(D) The correct answer is not here
\[ 10 \ln(2) - 3 \ln(10) = \ln(2)^{10} - \ln(10)^{3} \]
\[ = \ln \left( \frac{2^{10}}{10^{3}} \right) = \ln \left( \frac{1024}{1000} \right) \]

**Want to approx linearly**

\[ y = \ln(x) \]

\[ y - y_0 = m(x - x_0) \]
\[ (x_0, y_0) = (1, 0) \]
\[ y' = \frac{1}{x} \]
\[ y'(1) = 1 \]

\[ y - 0 = 1(x - 1) \]
\[ y = x - 1 \]

\[ y = 1.024 - 1 = 0.024 \]
Indeterminate Forms

\[ \frac{0}{0}, \infty, \infty - \infty, 0 \cdot \infty \]

\[ 1^0, \infty^0, 0^0 \]

Evaluate the following limit.

\[
\lim_{x \to \infty} \left(5e^{-x} + 1\right)^{2e^x} = \lim_{x \to \infty} e^{\ln \left(5e^{-x} + 1\right)^{2e^x}} = \lim_{x \to \infty} e^{2e^x \cdot \ln(5e^{-x} + 1)}
\]

(A) 10

(B) \(e^{10}\)

(C) \(e^{15}\)

(D) The correct answer is not here
If a box must have a square end then what dimensions will give the box of greatest volume which can be shipped via Priority Mail? The U. S. Postal Service will accept a box for shipment via Priority Mail only if the combined length and girth (distance around) is no more than 108 inches.

\[
\begin{align*}
\text{constraint Eqn} & \quad 4x + y = 108 \\
\text{Optimization Eqn} & \quad V = x^2y \\
y & = 108 - 4x
\end{align*}
\]

\[
V = x^2(108 - 4x) \\
V = 108x^2 - 4x^3
\]

\[
V' = 216x - 12x^2 \\
V'(x) = 0 \\
12x(18 - x) = 0 \\
x = 0, 18
\]

\[
\begin{array}{c|c|c|c}
\text{A) } & \text{B) } & \text{C) } & \text{D) } \\
18 \text{ by } 18 \text{ by } 36 \text{ inches} & 15 \text{ by } 15 \text{ by } 45 \text{ inches} & 20 \text{ by } 20 \text{ by } 32 \text{ inches} & \text{The correct answer is not here}
\end{array}
\]

\[
y = 108 - 4(18) = 36
\]
Choose a correct possible graph of \( y = f(x) \) below.

- [A]
- [B]
- [C]
- [D]