Inverse Trig Functions Examples
One Triangle Inverse Trig

1. For the problems below, draw triangles around the unit circle to answer the question. Be sure to use the domain/range of the inverse trig function to draw your triangle in the correct quadrant.

(a) Evaluate \( \cos^{-1}(\sqrt{3}/2) \).

\[
\begin{align*}
\cos^{-1}(\sqrt{3}/2) &= \theta \\
1/2 &= \cos(\theta)
\end{align*}
\]

What angle \( \theta \) (around the unit circle) between 0 and \( \pi \) gives us \( \cos(\theta) = \sqrt{3}/2 \)?

Our triangle is in quadrant I, where \( \cos(\theta) \) (the x-coordinate) is positive.

The \( \theta \) we want is \( \theta = \pi/6 \).

(b) Evaluate \( \arcsin(\sqrt{2}/2) \).

What angle \( \theta \) (around the unit circle) between \( -\pi/2 \) and \( \pi/2 \) gives us back \( \sqrt{2}/2 \) for \( \sin(\theta) \)?

(c) Evaluate \( \sin^{-1}(2) \).

Since the domain of \( \sin^{-1}(x) \) is between -1 and 1, this function is undefined.

(d) Evaluate \( \sec^{-1}(2/\sqrt{3}) \).

What angle \( \theta \) (around the unit circle) between 0 and \( \pi \) gives us back \( 2/\sqrt{3} \) for \( \sec(\theta) \)?

(\( \text{Remember: } \sec(\theta) = 1/\cos(\theta) = \text{hyp/adj} \).)
Inverse Trig on the Inside (One Triangle)

2. For the problems below, draw triangles around the unit circle to answer the question. Be sure to use the domain/range of the inverse trig function to draw your triangle in the correct quadrant.

(a) Evaluate \( \sin(\arctan(1/3)) \).

\[
\arctan(1/3) = \theta \\
tan(\theta) = 1/3
\]

What angle \( \theta \) (around the unit circle) between \(-\pi/2\) and \(\pi/2\) makes a triangle where \( \tan(\theta) = 1/3 \)?

Since \( \tan(\theta) = 1/3 = y/x \) is positive, we are in Quadrant I.

We use the Pythagorean Theorem to solve for \( r \).

\[
(3)^2 + (1)^2 = r^2 \\
10 = r^2 \\
r = \sqrt{10}
\]

(positive because \( r \) is the hypotenuse).

We use this triangle to complete the question:

\[
\sin(\arctan(1/3)) = \sin(\theta) = 1/\sqrt{10}
\]

(b) Evaluate \( \cos(\sin^{-1}(2/3)) \).

(c) Evaluate \( \tan(\cos^{-1}(2/3)) \).
Inverse Trig on the Outside (Two Triangle)

3. Evaluate $\sin^{-1}(\cos(2\pi/3))$.

We use the unit circle to draw a triangle and calculate $\tan(2\pi/3)$.

Using the special triangles on the front page, we get that $\cos(\theta) = \text{adj}/\text{hyp} = -1/2$.

Now we will calculate $\sin^{-1}(-1/2)$.

\[
\sin^{-1}(-1/2) = \theta \\
\sin(\theta) = -1/2
\]

What angle $\theta$ between $-\pi/2$ and $\pi/2$ makes $\sin(\theta) = -1/2$? Since $\sin(x)$ is negative (the y-value), we are in Quadrant IV.

So we conclude $\sin^{-1}(\cos(2\pi/3)) = \sin^{-1}(-1/2) = -\pi/3$

4. Evaluate $\cos^{-1}(\cos(-2\pi/3))$. 