The picture is "in motion".
Label each distance/length which is changing with a variable.

Interpret Problem info w/ your labeling
\[
\frac{dy}{dt} = 30, \quad \frac{dx}{dt} = 40
\]
Want \( \frac{dz}{dt} \) when \( t = 2 \)

Write an eqn fitting the geometry of the picture
\[
x^2 + y^2 = z^2
\]

Take \( \frac{d}{dt} \) of both sides of the eqn, and solve for desired quantity.
\[
\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(z^2)
\]

\[2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}\]

\[
\frac{dz}{dt} = \frac{2x \frac{dx}{dt} + 2y \frac{dy}{dt}}{2z}
\]

5. Use info given in problem to "plug in" for all needed quantities

\[
\frac{dx}{dt} = 40 \quad x = 80 \text{ after 2 hours}
\]

\[
\frac{dy}{dt} = 30 \quad y = 60 \text{ after 2 hours}
\]

\[
60^2 + 80^2 = z^2 \quad \text{when } x = 80, \ y = 60
\]

\[
z = 100
\]

\[
\frac{dz}{dt} = \frac{2(80)(40) + 2(80)(30)}{2(100)}
\]

\[
= 50 \text{ mph}
\]

Note: If the cars leave at different times, your answer will not end up as the Pythagorean Triple 30-40-50.
Example: A street light is mounted at the top of a 15 ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/sec. along a straight path. How fast is his shadow growing when he is 40 ft from the pole?

1. All parts "in motion" are labeled w/ variables.
   - The hypotenuse is not relevant, so it is not labeled.

2. Interpret Problem info w/ your labeling
   \[
   \frac{dx}{dt} = 5 \quad \text{Want } \frac{ds}{dt} \text{ when } x=40
   \]

3. Write an eqn \[ \text{Similar Triangles have equal side ratios} \]
   \[
   \frac{3}{6} = \frac{x+S}{15} \]
   \[
   \frac{1}{6} S = \frac{1}{15} x + \frac{1}{15} S
   \]
4. Take $\frac{d}{dt}$ of both sides of the eqn, solve

$$\frac{d}{dt} \left( \frac{1}{6} s \right) = \frac{d}{dt} \left( \frac{1}{15} x + \frac{1}{15} s \right)$$

$$\frac{1}{6} \frac{ds}{dt} = \frac{1}{15} \frac{dx}{dt} + \frac{1}{15} \frac{ds}{dt}$$

$$\frac{1}{6} \frac{ds}{dt} - \frac{1}{15} \frac{ds}{dt} = \frac{1}{15} \frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{\frac{1}{15} \frac{dx}{dt}}{\frac{1}{6} - \frac{1}{15}}$$

5. Use info to "plug in"

$$\frac{ds}{dt} = \frac{\frac{1}{15} (5)}{\frac{5}{30} - \frac{2}{30}} = \frac{\frac{1}{3}}{\frac{3}{30}} = \frac{1}{3} \left( \frac{30}{3} \right) = \frac{30}{9}$$

$$= \frac{10}{3} \text{ ft/sec}$$

Notes:
- We did not use $x=40$ in this question.
- If it had asked "how fast is the tip of his shadow moving" (i.e. away from the pole) a different labeling of the diagram is desired.

And solve for $\frac{dy}{dt}$
EX: A kite 100 ft above the ground moves horizontally at a speed of 8 ft/sec. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?

Label a diagram

\[ S \]

\[ \theta \]

\[ 100 \]

\[ x \]

\[ s \]

\[ \frac{dx}{dt} = 8 \quad \text{want} \quad \frac{d\theta}{dt} \quad \text{when} \quad s = 200 \]

We need an equation involving \( x \) and \( \theta \)

(\text{the variables whose rates are known or given, or wanted})

\[
\frac{d}{dt}(\tan \theta) = \frac{d}{dt}\left(\frac{100}{x}\right)
\]

\[
\sec^2 \theta \cdot \frac{d\theta}{dt} = -100x^{-2} \cdot \frac{dx}{dt}
\]
\[ \frac{d\theta}{dt} = -\frac{100}{x^2} \cdot \frac{dx}{dt} \cdot \sec^2\theta \]

Plug in: \( \frac{dx}{dt} = 8 \)

and find \( x \) and \( \theta \)
when \( s = 200 \)

\[ S = 200 \]

**Find \( x \)**

\[ \begin{align*}
X^2 + 100^2 &= 200^2 \\
x &= \sqrt{30,000} \\
x &= 100
\end{align*} \]

**Find \( \theta \)**

\[ \sin \theta = \frac{1}{2} \]
\[ \theta = \frac{\pi}{6} \]

\[ d\theta = \frac{-100}{(\sqrt{30,000})^2} \cdot 8 \]
\[ = \frac{-100}{30,000} \cdot \frac{8}{(\frac{2}{\sqrt{3}})^2} \]

\[ \sec \left( \frac{\pi}{6} \right) = \frac{2}{\sqrt{3}} \]
\[ = -\frac{8}{300} = \frac{3}{4} \]
\[ = -\frac{24}{1200} = -\frac{2}{100} \]
\[ = -0.02 \text{ rad/sec} \]
A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across the top and have a height of 1 ft. If the trough is being filled with water at a rate of 12 ft$^3$/min, how fast is the water level rising when the water is 6 inches deep?

\[
\frac{dV}{dt} = 12 \quad \frac{dh}{dt} \text{ when } h = \frac{1}{2} \text{ ft}
\]

We need an eqn involving $V$ & $h$  
(The variables whose rates are given or wanted)

\[
V = \left(\frac{1}{2} \times h\right) \times 10
\]

\[
V = 5 \times h
\]
* We must now substitute \( x_{\text{out}} \) 

Otherwise, after taking \( \frac{d}{dt} \) of both sides, we are left w/ \( \frac{dx}{dt} \) terms which we know nothing \( dt \) about

\[
\frac{x}{h} = \frac{3}{1}
\]

\( x = 3h \)

\[
V = 5 \,(3h) \, h
\]

\[
\frac{d}{dt} \left( V \right) = 15 \, h^2
\]

\[
\frac{dV}{dt} = 30h \cdot \frac{dh}{dt}
\]

\[
\frac{dh}{dt} = \frac{\frac{dV}{dt}}{30h} = \frac{12}{30} = \frac{2}{5} \approx 0.8 \text{ ft/min}
\]