Corner Point Theorem

Let \( z = ax + by \) be a linear function and 
\( R \) be a finite polygonal region in the plane.
Then, the min/max value of \( z \) over 
the region \( R \) is attained on the corner 
points of \( R \).

If your feasibility region is 
infinite, you must use lines of constancy.

Ex

Maximize \( z = x - 3y \) with constraints

\( 7x + 2y \leq 14 \)  
\( -13x + y \leq 3 \)  
\( y \geq 0 \)  
(The line \( y = 0 \))

Graph the lines which "border" the inequality

\( 7x + 2y = 14 \)  
\( 2y = -7x + 14 \)  
\( y = -\frac{7}{2}x + 7 \)  
\( x \)-int \( (2,0) \)  
\( y \)-int \( (0,7) \)

\( -3x + y = 3 \)  
\( x \)-int \( (-1,0) \)  
\( y \)-int \( (0,3) \)
2. Decide shading
   Test if (0,0) is shaded by the inequality
   (only works if your line does not pass through (0,0))

   \[ 7x + 2y \leq 14 \]
   \[ 0 \leq 14 \]
   We want to shade (0,0), so by the picture we know to shade below Line A

   \[ -13x + y \leq 3 \]
   \[ 0 \leq 3 \]

   Lines \[ y \geq 20 \]
   \[ R \] clearly shade above
3) Find corner points:
(-1, 0)
(2, 0)
Intersection of Lines A and B

We find using elimination

\[ 7x + 2y = 14 \]
\[ -3x + y = 3 \]

\[ \Rightarrow \]
\[ 7x + 2y = 14 \]
\[ 6x - 2y = -6 \]

\[ 13x = 8 \]
\[ x = \frac{8}{13} \]

Plug into get y-value

\[ y = \frac{63}{13} \]

Corner Points: (-1, 0) (2, 0) \( \left( \frac{8}{13}, \frac{63}{13} \right) \)
Since our shaded region is finite and polygonal, 
the min/max of our linear function 
will occur at one of these corner points.

To find the max of $z = x - 3y$ over this region, 
plug in the corner points.

$(-1,0) \rightarrow z = -1 - 0 = -1$
$(2,0) \rightarrow z = 2 - 0 = 2$
$(4/13, 63/13) \rightarrow z = 4/13 - 3(63/13) = \ldots$ some negative #

So our max for $z$ value is $z = 2$ at the point $(2,0)$. 
Ex: A company mines rubies & emeralds

Mine A produces 15 oz rubies per day at 5 oz emeralds a cost of $2200 per day

Mine B produces 10 oz rubies per day at 10 oz emeralds a cost of $2600 per day

The company must fill an order for 150 oz of rubies and 100 oz of emeralds in 25 days at minimum cost.

How can this be done?

What do we need to report back?

→ How many days each mine should run

→ What this "minimum cost" is.

What are your variables?

Your "unknowns" are usually what you are being asked to report back

x = # days to run Mine A
y = # days to run Mine B
What are your constraints?

i.e. write down the Linear Model

- Rubies: $15x + 10y \geq 150$
- Emeralds: $5x + 10y \geq 100$
  
  $x, y \geq 0$
  
  $x, y \leq 25$

Are you "optimizing" something?

We are minimizing cost

$\text{Cost} = C = 2200x + 2600y$

Feasibility Region

A) $15x + 10y = 150$
  
  x-int: $(10, 0)$
  
  y-int: $(0, 15)$

B) $5x + 10y = 100$
  
  x-int: $(20, 0)$
  
  y-int: $(0, 10)$

Shading:

- $15x + 10y \geq 150$
  
  $0 \geq 150$

- $5x + 10y \geq 100$
  
  $0 \geq 100$
Corner Points: (0, 25), (25, 0), (25, 25), (0, 15), (20, 0)

Intersection of Lines \( \alpha \) and \( \beta \)

Solve with Elimination:

\[
\begin{align*}
15x + 10y &= 150 \\
5x + 10y &= 100
\end{align*}
\]

\[10x = 50 \Rightarrow x = 5\]

\[15(5) + 10y = 150 \Rightarrow y = 7.5\]

Plug into \( y \)-value to get \( y \)-value:

\[(5, 7.5)\]

Optimize:

Since our feasibility region is finite, the minimum of the cost function will occur at a corner point.

Corner Points:

- \((5, 7.5)\)
- \((25, 0)\)
- \((0, 25)\)
- \((25, 25)\)
- \((0, 15)\)
- \((20, 0)\)

Plug into \( C = 2200x + 2600y \):

- \( C = 30,500 \)
- \( C = 55,000 \)
- \( C = 65,000 \)
- No way!
- \( C = 39,000 \)
- \( C = 44,000 \)
Running Mine A for 5 days & Mine B for 7.5 days fills the order at a minimum cost of $30,500.

(Is it ok to run a mine for 1/2 a day?)

Sure.

If not ok, we consider the corner point (5,8) in the list instead.
List of Pts separated by commas

29/3 not accepted

Extra line in solution to terrible region

Muncaster-5 ok

Lines go through the plane x-int & y-int may not

Randomization as new fixed

Tutoring starts Monday next