Instructions

- Write your name and UIN on top of this page.
- Please put your I-card face-up on your desk.
- Make sure you have the total of 6 pages, numbered from 1 to 6.
- You are not allowed to use your textbook, notebook, graphing calculator, cellphone, or smartphone.
- Read all the questions carefully before you answer them; point values vary.
- Show all your work. Remember, if I cannot tell how you arrived at your answers, you will not receive partial credit and you will not get full credit even if your answer is correct.
- You are expected to follow the university’s code of conduct. That means no cheating and you are not allowed to discuss this test outside of the class.
- You have 50 minutes to finish the exam.
- You can use the back of the test paper as scratch paper. If you need more, ask the instructor. Do Not use your own paper!
- Good luck!

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(1) Use the binomial theorem to evaluate
\[ \sum_{k=0}^{n} \frac{1}{k+1} \binom{n}{k}. \]
(2) Use the binomial theorem and the fact that

\[(1 + x)^a(1 + x)^b = (1 + x)^{a+b}\]

to prove the identity

\[\sum_{k=0}^{n} \binom{a}{k} \binom{b}{n-k} = \binom{a+b}{n}.\]

(Do not give a combinatorial proof.)
(3) Given $n$ chemists and $n$ biologists (assume that each person is either a chemist or a biologist, but not both), use inclusion-exclusion to obtain a summation formula for the number of ways to pair the $2n$ people as lab partners such that the $i$-th tallest chemist is not matched to the $i$-th tallest biologist. (Write the expression out.)
(4) Find the number of integral solutions of

\[ x_1 + x_2 + x_3 + x_4 = 17, \]

subject to \(0 \leq x_1 \leq 5, 2 \leq x_2 \leq 6, 0 \leq x_3 \leq 4\), and \(0 \leq x_4\).
(5) Use inclusion-exclusion to prove that
\[ \sum_{k=0}^{n} (-1)^k \binom{n}{k} (n-k)^n = n!. \]