This homework is due March 9 at noon. All answers must be fully justified. You must list all your sources and the names of the people you collaborated with.

1. Determine the number of permutations $i_1i_2i_3i_4i_5i_6i_7$ of $\{1, 2, 3, 4, 5, 6, 7\}$ that satisfy that $i_j \neq j$ if $j$ is an odd number.

2. Determine the number of permutations of the multiset $S = \{4 \cdot a, 5 \cdot b, 2 \cdot c\}$, where, for each type of letter, the letters of the same type do not appear consecutively. (For example abbbbacaca is not allowed, but abbbbacacab is.)

3. What is the number of ways to place six nonattacking rooks on the 6-by-6 board with forbidden positions as shown?

\[
\begin{array}{cccccc}
\times & \times & & & & \\
& \times & & & & \\
& & \times & \times & & \\
& & & & \times & \\
& & \times & \times & \times & \times \\
& & & & & \\
\end{array}
\]

4. Count the permutations $i_1i_2i_3i_4i_5i_6$ of $\{1, 2, 3, 4, 5, 6\}$, where $i_1 \neq 1; i_2 \neq 3; i_3 \neq 3; i_4 \neq 4; i_5 \neq 1, 3, 6$; and $i_6 \neq 6$.

5. A carousel has eight seats, each representing a different animal. Eight children are seated on the carousel but facing inward, so that each child faces another (each child looks at another boy’s front). In how many ways can the children change seats so that each faces a different child? How does the problem change if all the seats are identical?

6. Let $n$ be a positive integer and let $p_1, p_2, \ldots, p_k$ be all the different prime numbers that divide $n$. Consider the Euler functions $\phi$ defined by

$\phi(n) := |\{k : 1 \leq k \leq n, \gcd(k, n) = 1\}|$.

Use the inclusion-exclusion principle to show that

$\phi(n) = n \prod_{i=1}^{k} \left(1 - \frac{1}{p_i}\right)$.

7. The fixed points of a permutation $i_1i_2\cdots i_n$ of $\{1, 2, \ldots, n\}$ are the $i_j$ such that $i_j = j$. Prove that the number of permutations of $n$ where each fixed point is given a sign $+$ or $-$ is

$n! \left(1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}\right)$. 