This homework is due January 27, 2016 at noon. In order to obtain full marks, each solution should be clearly written with explanations for all claims made. You must list all your sources and the names of the people you collaborated with.

1. Count the number of four-digit numbers whose digits are either 1, 2, 3, 4, 5 such that:
   
   (a) The digits are distinct.
   (b) The number is even.
   (c) The digits are distinct and the number is even.
   (d) The digits are distinct or the number is even.

2. Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

3. How many ways are there to seat six people around a circular table where two seatings are considered the same when everyone has the same two neighbors without regard to whether they are right or left neighbors?

4. In how many distinguishable ways can the letters of “recurrence” be arranged?

5. (a) Find a minimum weight spanning tree for the following graph.

(b) What happens with the spanning tree you found in (a) when you apply the exchange property?

(c) Prove that if all the weights in a graph are distinct, then the minimum weight spanning tree is unique.