1. Show that if a group $G$ acts on a set $X$, then
   \[ R = \{ (x, gx) \mid x \in X, g \in G \} \]
   is an equivalence relation.
   What are the equivalence classes?

2. a) Prove that if $G \times X \to X$ is an action of a group $G$ on a set $X$ and $K \leq G$ is a subgroup, then the composite
   \[ K \times X \xrightarrow{\text{Id} \times \alpha} G \times X \xrightarrow{\alpha} X \]
   is an action of $K$ on $X$.

   b) Now assume $G$ is a L.C.H. top group, $K$ is a L.C.H. topological space, $K \leq G$ is closed and the action of $G$ on $X$ is proper. Prove that the action of $K$ on $X$ is proper as well.

   c) Prove if $G$ is L.C.H., then the action of $G$ on itself by left multiplication $G \times G \to G$, $(g, x) \mapsto gx$ is proper.

   d) Prove that the action of $\mathbb{Z}^n$ on $\mathbb{R}^n$ given by
   \[ z \cdot v = z + v \]
   is proper.

3. Consider the two point space $X = \{ 0, 1 \}$ with the topology
   \[ \mathcal{T} = \{ \emptyset, X, \{ 0 \} \}. \]
   Prove $X$ is path connected.

4. Prove that the product of path connected spaces is path connected.