Exercise 1.1. a. Let $M$ be a manifold, $V \subset M$ an open subset. Prove that if

$$\{\varphi_\alpha : U_\alpha \to \varphi_\alpha(U_\alpha) \subset \mathbb{R}^n\}$$

is an atlas on $M$ then $\{\varphi_\alpha|_{V \cap U_\alpha}\}$ is an atlas on $V$. Conclude that $V$ is a manifold.

b. Show that

$$GL(n, \mathbb{R}) := \{A \text{ an } n \times n \text{ matrix } | \det A \neq 0\}$$

is an $n^2$ dimensional manifold.

Exercise 1.2. Prove that for each matrix $A \in GL(n, \mathbb{R})$ the map

$$L_A : GL(n, \mathbb{R}) \to GL(n, \mathbb{R}), \quad L_A(B) := AB,$$

the left multiplication by $A$, is a diffeomorphism.

Exercise 1.3. Check that a notion of a smooth map between manifolds (as given in Lecture 2) does not depend on the choice of charts or atlases and therefore is well-defined.

Exercise 1.4. Define the complex projective space $\mathbb{C}P^{n-1}$ to be the set of complex lines through the origin in $\mathbb{C}^n$ and prove that it is a manifold. In particular define the underlying set of $\mathbb{C}P^{n-1}$ to be the quotient of $\mathbb{C}^n \setminus \{0\}$ by the equivalence relation $v \sim \lambda v$, $\lambda \in \mathbb{C}$, $\lambda \neq 0$.

What are the coordinate charts on $\mathbb{C}P^{n-1}$? What are the transition maps?