HW #4.4 Sketch of solution.

Since \( F : M \to N \) is a bijection, it has an inverse \( F^{-1} : N \to M \). The issue is whether \( F^{-1} \) is \( C^\infty \).

This is a local question. So pick a point \( q \in M \), a coordinate chart \( \psi : U \to \mathbb{R}^m \) with \( q \in U \), a coordinate chart \( \Psi : V \to \mathbb{R}^m \) on \( N \) with \( F(q) \in V \). Then

\[
d\psi_q, \ d\Psi_{F(q)} \text{ are isomorphisms.} \Rightarrow
\]

\[
d(\psi \circ F \circ \psi^{-1})_{\psi(q)} = d\psi_q \circ dF_{\psi(q)} \circ (d\psi^{-1})_{\Psi(F(q))} : \mathbb{R}^m \to \mathbb{R}^m
\]

is an isomorphism. Inverse function theorem \( \Rightarrow \)

\( \psi \circ F \circ \psi^{-1} \) is invertible with a \( C^\infty \) inverse on a small nbhd of \( \psi(q) \). Since \( (\psi \circ F \circ \psi^{-1})^{-1} = \psi \circ F^{-1} \circ \psi^{-1} \)

\( F^{-1} \) is \( C^\infty \) on a small nbhd of \( F(q) \). Since \( q \) is arbitrary, \( F^{-1} \) is \( C^\infty \).