1. Let $V, W$ be two finite dimensional vector spaces over the reals.
   a. Show that there is a natural isomorphism $\phi : V^* \otimes W^* \cong \text{Mult}(V, W; \mathbb{R})$ with
      $\phi(v^* \otimes w^*)(v, w) = v^*(v)w^*(w)$ for all $v^* \in V^*, w^* \in W^*, v \in V, w \in W$.
   a. Show that there is a natural isomorphism $\psi : V^* \otimes W^* \rightarrow (V \otimes W)^*$ with
      $\psi(v^* \otimes w^*)(v \otimes w) = v^*(v)w^*(w)$ for all $v^* \in V^*, w^* \in W^*, v \in V, w \in W$.

2. Let $V_1, V_2, V_3$ and $U$ be finite dimensional vector spaces over the reals. Show that for any multilinear map
   $\mu : V_1 \times V_2 \times V_3 \rightarrow U$
   there exists a unique linear map $\bar{\mu} : V_1 \otimes (V_2 \otimes V_3) \rightarrow U$ so that
   $\mu(v_1, v_2, v_3) = \bar{\mu}(v_1 \otimes (v_2 \otimes v_3))$
   for all $(v_1, v_2, v_3) \in V_1 \times V_2 \times V_3$.

3. Show that the map $\mathbb{R} \times V \rightarrow V, (a, v) \mapsto av$ gives rise to an isomorphism $\mathbb{R} \otimes V \cong V$ which
   sends $a \otimes v$ to $av$ for all $a \in \mathbb{R}$ and $v \in V$.

4. Suppose that $V$ is an $n$-dimensional vector space. Given a linear map $A : V \rightarrow V$, we get a map $\Lambda^n(A) : \Lambda^n(V) \rightarrow \Lambda^n(W)$, and since $\dim \Lambda^n(V) = 1$, the map $\Lambda^n(A)$ is multiplication by a scalar. Show that this scalar is $\det A$.  