1  Prove the mean value theorem for integrals: if $f : [a, b] \to \mathbb{R}$ is continuous, then there is $c \in (a, b)$ such that
\[
f(c) = \frac{1}{b-a} \int_a^b f.
\]

2  Show that the power series $\sum \frac{1}{n!} x^n$ converges on all of $\mathbb{R}$.

3  Find the radius of convergence of the power series $\sum n x^n$.

4  Suppose $f : [a, b] \to \mathbb{R}$ is continuous and $g : (c, d) \to [a, b]$ is differentiable. Prove that
\[
F(x) = \int_a^{g(x)} f
\]
is differentiable on $(c, d)$ and compute its derivative.

5  Let $S \subset \mathbb{R}$ be a set bounded above and $c > 0$. Prove that
\[
\sup \{cx \mid x \in S\} = c \sup \{x \mid x \in S\}.
\]

6  Prove that if $\sum |a_n|$ converges and $\{b_n\}$ is a bounded sequence then $\sum a_n b_n$ converges.