1. Suppose a group $G$ of order 10 acts on an infinite set $X$. What are all the possible sizes of the orbits of $G$? And how many orbits are there? Explain.
Now suppose $X$ has 8 elements. What are the possible sizes of orbits of $G$? Explain.
Now suppose $X$ has 11 elements. What's the minimal number of $G$ orbits in $X$? Explain.

2. Prove that the subgroup of $\mathbb{Z}$ generated by the set $\{n, m\}$ where $n, m > 0$ is $\gcd(n, m)\mathbb{Z}$.

3. Let $f : G \to L$ be a homomorphism, $g \in G$. Prove that the subgroup generated by $f(g)$ is isomorphic to $(g)/H$ where $H = \langle g \rangle \cap \ker f$. What does it tell you about the order of $f(g)$?

4. Prove that if $f : G \to H$ is an isomorphism and $g \in G$ has order $n < \infty$ than $f(g)$ also has to have order $n$. You may want to use problem 3 above or you may not.

5. Give an example of a group $G$ with a normal subgroup $K$ and $g \in G$, $k \in K$ such that $gkg^{-1} \not= k$. Hint: think of non-abelian finite groups you know.

6. Let $H, K$ be two normal subgroups of a group $G$ with $H \cap K = \{e\}$. Prove that $hk = kh$ for all $h \in H, k \in K$. Hint: consider $hkh^{-1}k^{-1}$. Does it have to be in $H$? in $K$?