1 Let $G$ be a group, $a \in G$.
   (i) Show that the map
   $$c_a : G \to G, \quad c_a(g) := aga^{-1}$$
   is a homomorphism.
   (ii) Show that $c_a$ is an isomorphism.

2 Suppose a group $G$ acts on a set $X$ and $f : H \to G$ is a homomorphism.
   Define the map $*: H \times X \to X$ by
   $$(h, x) \mapsto h * x := f(h) \cdot x$$
   where $\cdot$ denotes the action of $G$ on $X$. Show that $*$ is an action of $H$ on $X$.
   Hint: what do you need to check?

3 Let $G$ be a group, $H < G$ a subgroup and $g \in G$ an element. Consider
   the inversion map
   $$\text{inv} : G \to G, \quad \text{inv}(g) = g^{-1}.$$ 
   Prove that $\text{inv}(gH) = Hg^{-1}$. Hint: $\text{inv}(H) \subset H$ (why?) and $\text{inv} : H \to H$
   is a bijection
   Prove that $\text{inv}$ induces a bijection from the set of left cosets to the set of
   right cosets:
   $$\text{inv} : H \backslash G \to G/H.$$ 

4 Find all the elements in the subgroup of $S_4$ generated by the set $X = \{(12), (23)\}$. In other words what set is $\langle X \rangle$? Hint: id, $(12), (13), (12)(13)$
   are all in $\langle X \rangle$. Is there anything else?

5 Suppose $G$ is a group, $X$ is a set and $\varphi : G \to \text{Sym}(X)$ a homomorphism
   (as before $\text{Sym}(X)$ is the group of bijections of the set $X$). Prove that the map
   $$\alpha : G \times X \to X, \quad \alpha(g, x) := (\varphi(g))(x)$$
   is an action of $G$ on $X$. In other words, prove that $(g, x) \mapsto g \cdot x := (\varphi(g))(x)$
   is an action.

6 Consider the map $\exp : \mathbb{R} \to \mathbb{C}^\times := \mathbb{C} \setminus \{0\}$, $\exp(\theta) = e^{i\theta}$. It’s a homomorphism of groups: the group operation on $\mathbb{R}$ is $+$, the group operation on
   $\mathbb{C}^\times$ is multiplication. What is the image of $\exp$? What is the kernel of $\exp$?
   What are the left cosets of $\ker(\exp)$ in $\mathbb{R}$?

7 Consider the action of $\mathbb{Z}$ on $\mathbb{R}$ given by
   $$n \cdot x := (-1)^n x.$$ 
   What are the orbits of this action? What is the stabilizer of $\pi \in \mathbb{R}$? What is the stabilizer of 0?