1. Prove that the ideal $I = (2, 1 - \sqrt{-5})$ in the ring $\mathbb{Z}[\sqrt{-5}]$ is maximal. Prove that it is not principal.

2. Recall that every ideal $I \subset \mathbb{Z}$ is principal. Consider the ideals $6\mathbb{Z}$ and $8\mathbb{Z}$ in the ring $\mathbb{Z}$. Find $m, n, k \in \mathbb{Z}$ so that 
   
   $$m\mathbb{Z} = 8\mathbb{Z} \cap 6\mathbb{Z}, \quad n\mathbb{Z} = 8\mathbb{Z} + 6\mathbb{Z}, \quad k\mathbb{Z} = (8\mathbb{Z})(6\mathbb{Z}).$$

3. Let $R$ be a ring with a subring $A$ and let $I \subset R$ be an ideal. Prove that 
   
   1. $A + I := \{a + i \in R \mid a \in A, i \in I\}$ is a subring of $R$.
   2. $A \cap I$ is an ideal in $A$.
   3. $I$ is an ideal in $A + I$.
   4. $(A + I)/I$ is isomorphic to $A/(A \cap I)$.

4. Let $R$ be a ring, $K, I \subset R$ ideals with $K \subset I$. Prove that $I/K := \{i + K \mid i \in I\}$ is an ideal in $R/K$ and that $(R/K)/(I/K)$ is isomorphic to $R/I$.

5. Let $F$ be a field and $p(x) \in F[x]$ be a polynomial of degree 3. Prove that $p$ is irreducible if and only if $p$ has no roots in $F$.

6. Suppose a ring $R$ is a UFD and $\{a_i\}_{i=1}^{\infty}$ is a sequence of elements in $R$ with $(a_i) \subset (a_{i+1})$ for all $i$. Prove that the increasing chain of ideals $(a_1) \subset (a_2) \subset \ldots$ eventually stabilizes. That is, show that there is $m \in \mathbb{N}$ so that $(a_{m+k}) = (a_m)$ for all $k \in \mathbb{N}$. Hint: write $a_1$ as a product of irreducibles/primes. $(a_1) \subset (a_n)$ means that $a_n | a_1$. Therefore ...

7. Prove that the quotient ring $\mathbb{R}[x]/(x^2 + 2)$ is isomorphic to the field of complex numbers.