Let $V$ and $W$ be two finite dimensional vector spaces and $L(V,W)$ the vector space of linear maps from $V$ to $W$.

(a) Prove: $\dim L(V,W) = (\dim V)(\dim W)$.

(b) If $\{e_i\}_{i=1}^m$ is a basis of $V$ and $\{f_j\}_{j=1}^n$ is a basis of $W$,
what is a corresponding basis of $L(V,W)$?

Hint: If $v \in V$ and $w \in W$, we have a linear map

$$e \circ w : V \to W, \quad (e \circ w)(v) = e(v)w$$

for all $v \in V$.

#2 Prove that if $M \subset \mathbb{R}^k$ and $N \subset \mathbb{R}^l$ are manifolds then so is $M \times N \subset \mathbb{R}^{k+l}$.

#3 Prove that determinant $\det : \mathbb{R}^n \to \mathbb{R}$ is differentiable at $I$ and that $D \det (I)A = \text{tr} A$, where $I$ is the identity matrix and $\text{tr}(a_{ij}) = \sum_{i=1}^n a_{ii}$ is the trace.

#4 Let $Q = A \times B$, where $A = [0,1]$ and $B = [0,1]$. Find an example of a bounded function $Q \to \mathbb{R}$ so that $\int_Q f$ exists, $\int_{x \in A} \int_{y \in B} f(x,y)$ exists but $\int_{y \in B} \int_{x \in A} f(x,y)$ does not.

#5 Construct a sequence of functions $f_n \in C^0([0,1])$ so that $\int_{[0,1]} f_n = 1$ and $\lim_{n \to \infty} f_n(x) = 0$ for all $x \in [0,1]$. 