1 Given a regular curve $c : [0, 1] \to \mathbb{R}^n$ how do you find a change of parameter $t = \varphi(s)$ so that $c(\varphi(s))$ is a unit speed curve?

2 What is the length of a curve $c : [a, b] \to \mathbb{R}^n$. If we apply a parameter transformation, does the length change?

3 What is the curvature of the plane curve $c(t) = (t, e^t)$?

4 What does it mean for a curve to be periodic? What is a winding number of a periodic curve? Can the winding number be negative? zero? an irrational number? a rational number? Does it depend on a choice of a parametrization?

5 What does it mean for a curve to be simple? Closed? What do you know about winding numbers of simple closed curves? What is the winding number of $c(t) = (2 \cos t, 3 \sin t)$?

6 What does the Four vertex theorem say? Does it apply to the curve that is given in polar coordinates by $(\theta(t), r(t)) = (t, 1 - 2 \sin t)$?

7 If $G$ is a region in $\mathbb{R}^2$ whose boundary $\partial G$ is a simple closed curve, can we have

$$\text{Area } (G) > \frac{1}{4\pi} (\text{length}(\partial G))^2.$$ 

If Area $(G) = \frac{1}{4\pi} (\text{length}(\partial G))^2$, what do you know about the shape of $G$?

8 What is the Frenet apparatus of a space curve? How do you compute it? What are the Frenet equations?

9 If $\kappa : [a, b] \to (0, \infty)$ and $\tau : [a, b] \to \mathbb{R}$ are two smooth functions, does there exist a curve $c(t)$ with curvature $\kappa$ and torsion $\tau$?

10 If $c_1, c_2 : [a, b] \to \mathbb{R}^3$ are two regular curves with the same curvature and torsion, how are they related? Are they the same? differ by a rotation? differ by a translation?

11 Prove that if the curvature of a unit speed curve $c$ is identically zero, then the curve is a straight line.

12 Prove that if the torsion of a space curve is identically zero, then the curve lies in a plane.

13 Is the set $\{(x, y, z) \mid z^2 > x^2 + y^2\}$ open in $\mathbb{R}^3$?

14 Is the set

$$S = \{(x, y, z) \mid z^2 = x^2 + y^2\}$$

a regular surface?

15 What is the tangent plane to the surface $z = x^2 + y^2$ at the point $(1, 1, 2)$?

16 Is the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$ orientable? If yes, why? If no, why not?
17 Let $c : (0, 1) \to \mathbb{R}^3$ be a unit speed curve. Prove that

$$F : (0, 1) \times (0, 1) \to \mathbb{R}^3, \quad F(u, v) = c(u) + c(v)$$

is not a local parametrization of a surface.

18 Suppose $F : \mathbb{R}^3 \to \mathbb{R}^3$ is a smooth map such that the Jacobian $DF(0, 0, 0)$ is invertible. Is $F$ necessarily invertible in a neighborhood of $(0, 0, 0)$? If yes, why? If no, why not?

19 Prove that the graph

$$S = \{(x, y, z) \mid z = f(x, y)\}$$

of a smooth function $f : \mathbb{R}^2 \to \mathbb{R}$ is an orientable regular surface. What is the tangent space to $S$ at a point $(a, b, f(a, b))$?