1. Consider the parametrization of the sphere given by

\[ F(\theta, \varphi) = (R \sin \varphi \cos \theta, R \sin \varphi \sin \theta, R \cos \varphi). \]

Show that the first fundamental form with respect to this parametrization is

\[ R^2 \sin^2 \varphi \, d\theta \otimes d\theta + R^2 \, d\varphi \otimes d\varphi. \]

2. Recall that the graph of a function \( f(x, y) \) can be parametrized by

\[ F(x, y) = (x, y, f(x, y)). \]

Show that the first fundamental form with respect to this parametrization is

\[ (1 + \left(\frac{\partial f}{\partial x}\right)^2) \, dx \otimes dx + \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \, dx \otimes dy + \frac{\partial f}{\partial y} \frac{\partial f}{\partial x} \, dy \otimes dx + (1 + \left(\frac{\partial f}{\partial x}\right)^2) \, dy \otimes dy \]

3. As in problem 2 above, let \( S \) be the graph of a function \( f(x, y) \). Compute the unit normal \( N \) to the surface \( S \) in terms of the partial derivatives of \( f \).

4. For each of the following surfaces \( S \) find the unit normal map (also called the Gauss map)

\[ N : S \to S^2. \]

In particular describe the image \( N(S) \) in \( S^2 \).

a. \( S \) is the cone minus the origin \( \{(x, y, z) \mid z^2 = x^2 + y^2, (x, y, z) \neq (0, 0, 0) \} \). What happens as you approach the origin?

b. \( S \) is the plane \( \{(x, y, z) \mid x + y + z = 0\} \).

c. \( S \) is the sphere \( \{(x, y, z) \mid (x - 1)^2 + y^2 + (z + 2)^2 = 1\} \).

d. \( S \) is the saddle \( \{(x, y, z) \mid z = xy\} \).