1. Find a pair of real numbers $a, b$ so that

\[ \frac{1}{2 + 3i} = a + ib. \]

2. Prove that for any two complex numbers $z = x + iy, w = u + iv$ we have

\[ \overline{z \cdot w} = \overline{z} \cdot \overline{w}. \]

3. Use the fact that $e^{i \theta_1} e^{i \theta_2} = e^{i(\theta_1 + \theta_2)}$ to deduce the addition formula for cosine:

\[ \cos(\theta_1 + \theta_2) = \cdots \]

4. Find a sequence of complex numbers $\{c_k\}_{k \in \mathbb{Z}}$ so that

\[ \sin \theta = \sum_{k=-\infty}^{\infty} c_k e^{ik\theta}. \]

5. Draw a picture of the plane curve

\[ \alpha(\varphi) = (1 - 2 \sin(\varphi))(\cos \varphi, \sin \varphi)^T. \]

Find the vertices of the curve (hint: use the formula from exercise 2.10 on p. 36 to compute curvature). Does your answer contradict the four vertex theorem (theorem 2.2.18)? Explain.