Homework #2, Math 416, **Prof. Eugene Lerman**
Due Wednesday, January 30, 2019 in class

Notation $(x_1, x_2, \ldots, x_n)^T := \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n.$

Solve the following problems:

1 Let $V = \{(x_1, x_2)^T \in \mathbb{R}^2 \mid x_1, x_2 \geq 0\}$. Define the operations $+ : V \times V \to V$ by $(x_1, x_2)^T + (y_1, y_2)^T := (x_1 + y_1, x_2 + y_2)^T$ and $\cdot\!: \mathbb{R} \times V \to V$ by $\lambda \cdot (x_1, x_2)^T := (\lambda^2 x_1, \lambda^2 x_2)^T$. Is V a vector space with these operations? Prove your answer.

2 Let $V = (0, \infty)$, Define addition $\oplus : V \times V \to V$ by $a \oplus b := ab$ and scalar multiplication $\odot : \mathbb{R} \times V \to V$ by $\lambda \odot a := a^\lambda$. Is V a vector space with these operations? Prove your answer.

3 Is the vector $(-1, 2, 3)^T \in \mathbb{R}^3$ in the span of the set $\{(3, 4, 2)^T, (1, 3, 3)^T\}$? Prove your answer.

4 Suppose V is a vector space , $n \geq 1$, $\{v_1, \ldots, v_n\}$ is a subset of V and v is in span$\{v_1, \ldots, v_n\}$. Prove that the set $\{v, v_1, \ldots, v_n\}$ is linearly dependent.

5
(a) Let V be a vector space, $U, W \subset V$ two subspaces. Prove that their intersections $U \cap W$ is also a subspace of V.
Hint: don't forget to check that $U \cap W \neq \varnothing$.

(b) Now generalize: let $\mathcal{S}$ be a collection of subspaces of V. Prove that the intersections $\bigcap_{U \in \mathcal{S}} U$ of all subspaces in $\mathcal{S}$ is a subspace of V.

6 Let V be an n-dimensional vector space.
(a) Prove that if a subset S of V spans V then S has at least n elements. Hint: Lemma 4.1 and Theorem 4.3 from lecture 4 may be useful.

(b) Prove that if $S \subset V$ is linearly independent then S has at most n elements.

7 Let V be a vector space, S, S' two subsets of V with $S' \subset S$. Prove that span $S' \subset$ span S.

8 Let V be a vector space, $S \subset V$ a nonempty subset and $v \in$ span S. Prove that span $S =$ span $S \cup \{v\}$.