Homework #1, Math 416, **Prof. Eugene Lerman**
Due Wednesday, January 23, 2019 in class

Sections 1.1 – 1.3 of Treil's *Linear algebra done wrong* (it's not done wrong, don't worry).
Solve the following problems:

1 Which of the following sets with natural addition and multiplication by scalars are vector spaces? Justify your answer briefly.

- The set of continuous real-valued functions on the real line.
- The set of all non-negative (real-valued) functions on the interval $[0, 1]$.
- The set of polynomials with real coefficients of degree exactly n.

2 Find all solutions (if any) of the system

$$\begin{cases} x_1 + 3x_2 + 2x_3 &= 2 \\ x_1 + 6x_2 + x_3 &= 3 \\ 2x_1 + 3x_2 + 5x_3 &= 5. \end{cases}$$

3 Let $C(\mathbb{R})$ denote the vector space of all continuous functions $f\colon \mathbb{R} \to \mathbb{R}$. Let $T\colon C(\mathbb{R}) \to \mathbb{R}$ be the transformation (that is, a function or a map) defined by

$$T(f) = \int_1^2 f(x)\, dx.$$

Is T linear? Prove your answer.

4 Multiply a vector by a matrix as indicated:

a. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} =$

b. $\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} =$

c. $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} =$

5 a Find all solutions (if any) of the system

$$\begin{cases} x_1 + 2x_2 - x_3 + 2x_4 = 3 \\ 3x_1 + 7x_2 + 5x_4 = 8 \\ -x_1 + 7x_3 - 2x_4 = -1 \end{cases}.$$

(part **b** is on the next page)

b If we change the right hand side $\begin{bmatrix} 3 \\ 8 \\ -1 \end{bmatrix}$ to some other vector $\vec{b} \in \mathbb{R}^3$, would the system necessarily have a solution? If so, justify; if not, find an explicit right-hand side $\vec{b} \in \mathbb{R}^3$ such that the system has no solution.