Review for first midterm, Math 416 E1, Prof. Lerman  Monday, February 27, 2012

The first midterm will cover sections 1.1–1.7 and 2.1–2.7.

**Important concepts:** vector space, linear map, linear independence, spanning set, finite dimensional vector space, basis, dimension, subspace, row reduction and elementary matrices, pivots, range and kernel of a linear map and of a matrix, rank of a matrix, invertible matrices and linear maps.

**Skills**

- finding particular and general solutions of systems of linear equations;
- checking that a given collection of vectors is linearly independent, spans, is a basis;
- checking that a matrix is invertible, computing the inverse of an invertible matrix,

**Statements of theorems to know**

- a finite dimensional vector space has a basis;
- any two bases (of a finite dimensional vector) space have the same number of vectors;
- any spanning set (of a finite dimensional vector space) contains a basis;
- any linearly independent set (in a finite dimensional vector space) can be completed to a basis;
- If $V$ is a finite dimensional vector space and $U \subset V$ is a proper subspace, then $\dim U < \dim V$.
- Theorem 6.1 on p. 56 — form of a general solution of a inhomogeneous equation $Ax = b$;
- the rank theorem — theorem 7.1 on p. 63;
- the “other” rank theorem — theorem 7.2 on p. 64.

I expect you to be able to write down short proofs of statements such as:

- Prove that if $X, Y$ are two subspaces of a vector space $V$ then so is their intersection $X \cap Y$.
- Prove that the composit of two linear maps is again a linear map.
Any homework problem is a fair game (except 3.3 c, d). There may be a problem on the material of 2.7 similar to problems 7.1, 7.2 or 7.3 on pp. 66–67.

The general format of the exam will be: a few true/false questions, a proof, some computational problem(s), a conceptual problem.