Read section 1 (Fundamentals) of Lax’s book

Solve the following problems:

1. Which of the following sets with natural addition and multiplication by scalars are vector spaces. Justify your answer.
   - The set of continuous functions on the interval $[0, 1]$.
   - The set of all non-negative functions on the interval $[0, 1]$.
   - The set of polynomials of degree exactly $n$.

2. Evaluate each statement as true or false. Justify your answer.
   - Any set of vectors in a vector space containing the zero vector $\vec{0}$ is linearly dependent.
   - A basis of a vector space must contain the zero vector.
   - A basis of a vector space may not contain the zero vector.
   - A subset of a linearly independent set of vectors is linearly independent.
   - A subset of linearly dependent set of vectors is linearly dependent.

3. Prove that if $V$ is a vector space and $Y, Z \subset V$ are subspaces then $Y \cap Z$ is also a subspace.