Which of the following relations are equivalence relations? Justify your answer. That is, check the three conditions.

a. Fix an integer $n \neq 0$. Define a relation $\sim$ on $\mathbb{Z}$ by $x \sim y$ if and only if $n | (x - y)$.

b Define a relation $\sim$ on $\mathbb{Z}$ by $a \sim b$ if and only if $a | b$.

c Define a relation $\sim$ on $\mathbb{R}^2$ by $(a, b) \sim (a', b')$ if and only if $a + b = a' + b'$.

d Define a relation $\sim$ on $\mathbb{R}^2$ by $(a, b) \sim (a', b')$ if and only if $2a^2 + b = 2(a')^2 + b'$.

e Define a relation $\sim$ on $\mathbb{R}^2$ by $(a, b) \sim (a', b')$ if and only if $a = a'$.

f Fix a polynomial $d(x)$. Define a relation $\sim$ on $\mathbb{R}[x]$ by $f \sim g$ if and only if $d | (f - g)$.

Recall that given an equivalence relation $\sim$ on a set $X$, the equivalence class of $x \in X$ is the set

$$[x] = \{ y \in X | x \sim y \}.$$ 

a Let $X = \mathbb{R}^2$ and $\sim$ the equivalence relation defined by $(a, b) \sim (a', b')$ if and only if $b = b'$. What is the equivalence class of $(1, 1)$? Draw a picture of the class.

b Let $X = \mathbb{Z}$ and $\sim$ the equivalence relation defined by $a \sim b$ if and only if $3 | (b - a)$. What are the equivalence classes of 1, 10, 11 and 12? Can it happen that $a \neq b$ and $[a] = [b]$?

c Let $X = \mathbb{R}^2$ and $\sim$ the relation defined by $(a, b) \sim (x, y)$ if and only if $a^2 + b^2 = x^2 + y^2$. What is the equivalence class of $(1, 0)$? Draw a picture of the class.

3 Let $J = \{ 3x + 2 \mid x \in \mathbb{Z} \}$. Is $J$ an ideal of $\mathbb{Z}$? Explain.