MATH 347 Practice problems for 2nd mid-term. (4/2/08)

Note Title

#1 a) Write $\frac{2+3i}{7-i}$ in the form $a+ib$, $a,b \in \mathbb{R}$

b) $|1-i| =$

c) $e^{i\pi/3} = a+ib$ for some $a,b \in \mathbb{R}$. What are they?

#2 Compute $\lim_{n\to\infty} \frac{n^2+2n+3}{2n^2-n}$. Justify all the steps.

#3 The sequence $a_n = (-1)^n + 1$ Cauchy? Prove your answer.

#4 Prove with only the definition of the limit: if $\{a_n\}, \{b_n\} \subseteq \mathbb{C}$ $a_n \to L$, $b_n \to M$ then $a_n + b_n \to L+M$.

#5 Which of the following series converge? Explain.

a) $\sum_{n=1}^{\infty} \frac{1}{n}$ b) $\sum_{n=0}^{\infty} \frac{3^n}{n!}$ c) $\sum_{n=1}^{\infty} (1+\frac{1}{n})$ d) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

#6 Does the sequence $a_n = \sin(n)$ have a convergent subsequence?

#7 For what values of $z \in \mathbb{C}$ is the series $\sum_{n=0}^{\infty} \frac{(-1)^n z^{n+1}}{n+1}$ guaranteed to converge? What theorem is relevant?

#8 Prove that if $\{z_n\} \subseteq \mathbb{C}$ converges then $\exists M > 0$ so that $|z_n| < M$ for all $n \in \mathbb{N}$

#9 Prove that $\sup([0,1)) = 1$

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#10 Find $p, q \in \mathbb{Z}$ so that $p/q = 0.888\ldots$

#11 Does the sequence $a_n = \frac{1}{n}\sin(n)$ converge? Explain.

#12 a) Prove that $|z+w| \geq ||z|-|w||$ for all $z, w \in \mathbb{C}$

Hint: $|z| = |z+w-w| \leq |z+w| + |w|$ and $|w| \leq |z+w| + |z|$

b) Prove that if $\{z_n\} \subseteq \mathbb{C}$ converges to L then $|z_n| \to |L|$. Is the converse true? Explain.