

- #1 a) Prove that the sequence $a_n = \frac{1}{n} + (-1)^n$ is bounded.
 b) Does it have a convergent subsequence? If it does, produce one.
 c) Is $\{a_n\}$ Cauchy? Prove your answer.

#2 Show that the series $\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{2}}$ diverges.
 Hint: comparison test.

#3 Prove that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges.

Hint:

$$\begin{aligned} s_n &= \sum_{k=1}^n \frac{1}{k(k+1)} = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{k \cdot (k+1)} = \\ &= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \dots + \left(\frac{1}{k} - \frac{1}{k+1}\right) = \\ &= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{k} - \frac{1}{k+1} = \dots \end{aligned}$$

#4 Prove that $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges.