MATH 347 Homework #7 Due 3/14/08

Note Title

1. Given a sequence $\{a_n\}$ and given $k \in \mathbb{N}$, let $\{b_n\}$ be the sequence defined by $b_n = a_{n+k}$ $(\forall n \in \mathbb{N})$. That is, the terms of $\{b_n\}$ are the same as the terms of $\{a_n\}$ afer the first k terms have been skipped. Prove that $\{b_n\}$ converges $\Leftrightarrow$ $\{a_n\}$ converges, and if they converge show that $\lim a_n = \lim b_n$. Thus converges of a sequence is not affected by ommitting finitely many terms.

2. a) Prove that if $\lim a_n = L$ then $\lim |a_n| = |L|$
   b) Suppose $\lim |a_n| = |L|$. Does it follow that $a_n \to L$?
   c) Prove: $\lim a_n = 0 \Leftrightarrow \lim |a_n| = 0$.

3. A sequence $\{z_n\}$ of complex numbers converges to $L \in \mathbb{C}$ iff $\forall \varepsilon > 0$ $\exists N \in \mathbb{N}$ so that $|z_n - L| < \varepsilon$ for all $n \in \mathbb{N}$.

a) Prove that if $z_n \to L$ and $w_n \to M$ then $z_n + w_n \to L + M$ for any two sequences $\{z_n\}$ and $\{w_n\}$ of complex numbers.

b) Prove that if $\{z_n\}$ is a sequence of complex numbers and if for some $c \in \mathbb{C}$, $z_n = c$ for all n, then $z_n \to c$.

4. A sequence $\{z_n\}$ of complex numbers is bounded iff $\exists M > 0$ so that $|z_n| < M$, $\forall n \in \mathbb{N}$. Prove: if a sequence $\{z_n\}$ of complex numbers converges, then it is bounded.