Note Title

MATH 347, Homework #4   Due 2/15/08

#1 Let $p(x), g(x), f(x)$ be polynomials. Suppose $f(x) \mid p(x)$ and $f(x) \mid g(x)$. Prove that for <u>any</u> polynomials $a(x), b(x)$, $f(x) \mid (a(x)p(x) + b(x)g(x))$.

#2 A subset $I \subseteq \mathbb{Z}$ is an <u>ideal</u> iff (1) $a \in I$ and $b \in I \Rightarrow a+b \in I$.
and $a \in I$, $d \in \mathbb{Z} \Rightarrow ad \in I$.
Prove that for any ideal I there is an integer $a \geq 0$ so that

$$I = \{ax \mid x \in \mathbb{Z}\} \equiv a\mathbb{Z}.$$

<u>Hint:</u> division algorithm

#3 (a) Prove that for any $a, b \in \mathbb{Z}$ the set

$$(a,b) := \{au + bv \mid u, v \in \mathbb{Z}\}$$

is an ideal.

(b) Prove that $(a,b) = \{dx \mid x \in \mathbb{Z}\} \equiv d\mathbb{Z}$ where $d = \gcd(a,b)$.

#4 Prove that if p is a prime number then $\sqrt[3]{p}$ is irrational.

#5 Do there exist integers x and y so that $13x + 17y = 132$?
<u>Hint</u> #1 from Homework #3 may be good for something.