1. Prove that for any two sets $A$ and $B$

\[ A \cap B \subset A \quad \text{and} \quad B \subset A \cup B. \]

2. (i) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$ for any three sets $A, B$ and $C$.

(ii) Prove that $A \cup \emptyset = A$ for any set $A \neq \emptyset$.

(It’s also true if $A = \emptyset$ but I am not asking you to prove it.)

(iii) Suppose $A, B$ are two subsets of a set $X$. Prove that

\[ (A \cap B)^c = A^c \cup B^c, \]

that is, that the complement of an intersection is the union of the complements.

3. Let $X$ be a (nonempty) set. Show that

\[ \mathcal{R} := \{(x, y) \in X \times X \mid x = y\} \]

is an equivalence relation on the set $X$.

4. Show that

\[ \mathcal{R} := \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x \leq y\} \]

is a reflexive and transitive relation on the set $\mathbb{R}$ of real numbers. Prove that $\mathcal{R}$ is not symmetric. Is $\mathcal{R}$ an equivalence relation? Explain.

5. Since the empty set $\emptyset$ is a subset of every set, for any set $X$ we have $\emptyset \subset X \times X$. Prove that for any nonempty set $X$,

\[ \emptyset \subset X \times X \]

is not an equivalence relation.