#1 a). Write $\frac{2+3i}{7-i}$ in the form $a+ib$, $a, b \in \mathbb{R}$

b) $|1-i| =$

c) $e^{i\pi/3} = a+ib$ for some $a, b \in \mathbb{R}$. What are they?

#2 $\lim\limits_{n \to \infty} \frac{n^2+2n+3}{2n^2-n} =$

#3 Is the sequence $a_n = (-1)^n + 1$ Cauchy? Prove your answer.

#4 Prove using only the definition of the limit: if $a_n \to L$, $b_n \to M$ then $a_n + b_n \to L + M$.

#5 a. Prove $f(x) = 3\sin x$ is continuous at 0. Hint: $|\sin x| \le |x|$ $\forall x$.

b. $\lim\limits_{n \to \infty} \sin(\frac{1}{n}) =$

#6 Which of the following series converge? Explain.

a) $\sum\limits_{n=1}^{\infty} \frac{1}{n}$ b) $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{n}$ c) $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$ d) $\sum\limits_{n=0}^{\infty} \frac{3^n}{n!}$ e) $\sum\limits_{n=1}^{\infty} (1+\frac{1}{n})$

#7 Does the sequence $a_n = \sin(n)$ have a convergent subsequence?

#8 For what values of $z \in \mathbb{C}$ the series $\sum\limits_{n=0}^{\infty} \frac{(-1)^n z^{n+1}}{n+1}$ is guaranteed to converge? What theorem is relevant?

#9 Is the function $f(x) = \begin{cases} 1/x & x \neq 0 \\ 1 & x = 0 \end{cases}$ continuous at 0? Justify your answer using relevant theorems.

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#10 a) Prove that if $f: A \to B$ and $g: C \to D$ are onto, then so is $h: A \times C \to B \times D$, $h(a,b) = (f(a), g(b))$.

b) Prove that $\mathbb{Q} \times \mathbb{Q}$ is countable.

#11 Let $f: A \to B$ be a bijection and be $f^{-1}: B \to A$ its inverse. Prove that f^{-1} is a bijection. What is the inverse of f^{-1}?

#12 Prove that the set of irrational numbers is not countable.