Homework 9, Math 347, **Prof. Eugene Lerman**
Due Friday, October 12, 2007

1 Definition: A function $f : \mathbb{C} \to \mathbb{R}$ is *continuous* at $L \in \mathbb{C}$ if for every $\epsilon > 0$ there is $\delta > 0$ so that

$$|z - L| < \delta \Rightarrow |f(z) - f(L)| < \epsilon.$$

Prove, using the definition above, that function $f(z) = |z|$ is continuous at all points of $\mathbb{C}$

2 Prove that if $f : \mathbb{C} \to \mathbb{R}$ is continuous and $\{z_n\} \subset \mathbb{C}$ converges to L then $\{f(z_n)\}$ converges to $f(L)$.

3 Prove that a function $f : \mathbb{R} \to \mathbb{R}$ is constant, then it is continuous.

4 Prove that if $f, g : \mathbb{R} \to \mathbb{R}$ are continuous (at all points of $\mathbb{R}$) then so are their sum $f + g$ and product fg.

Hint: Let $\{x_n\}$ be a sequence converging to L. Then $f(x_n) \to f(L)$, $g(x_n) \to g(L)$ and $f(x_n) + g(x_n) \to ...$

5 Find the mistake in the "proof" of the following bogus
Claim any continuous function $f : (0, 1) \to \mathbb{R}$ is bounded.
Proof: Suppose f is not bounded. Then for any $n \in \mathbb{N}$ there is $x_n \in (0, 1)$ with $|f(x_n)| \geq n$. Since $0 < x_n < 1$, $\{x_n\}$ is bounded. Hence it has a convergent subsequence $\{x_{n_k}\}$. Let $L = \lim_{k\to\infty} x_{n_k}$. Since f is continuous $f(x_{n_k}) \to f(L)$. On the other hand $|f(x_{n_k})| \geq n_k$ by construction. Hence $\{f(x_{n_k})\}$ is unbounded and cannot converge. Contradiction.

Hint: $f(x) = \frac{1}{x}$ is continuous on $(0, 1)$ but is not bounded.