Homework 8

Due November 2, 07 in class.

Note Title

#1 Prove that the sequence $a_n = \frac{1}{n} + (-1)^n$ is bounded. Does it have a convergent subsequence? If it does, produce one.

#2 A sequence $\{a_n\}$ is <u>contractive</u> if there is a constant k with $0 < k < 1$ such that

$$|s_{n+2} - s_{n+1}| < k\,|s_{n+1} - s_n| \quad \text{for all } n \in \mathbb{N}.$$

Prove that every contractive sequence is a Cauchy sequence hence is convergent. <u>Hint</u>

$$|s_3 - s_2| < k\,|s_2 - s_1|$$
$$|s_4 - s_3| < k\,|s_3 - s_2| < k^2\,|s_2 - s_1| \;\ldots$$

#3 Use the comparison test to show that the series $\sum_{n=2}^{\infty} \frac{1}{n - \sqrt{2}}$ is divergent.

#4 Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges.

<u>Hint:</u>

$$s_n = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)} = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$$
$$= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \cdots - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1} =$$

#5. Prove (without the use of calculus) that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. <u>Hints:</u> (1) $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \sum_{n=2}^{\infty} \frac{1}{n^2} = 1 + \sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$

(2) $\frac{1}{(n+1)^2} \le \frac{1}{n(n+1)}$ for all $n \in \mathbb{N}$

#6 Prove that $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges.