MATH 347 Homework #5 Due 10/12/07

Note Title

1. Prove that for any $z, w \in \mathbb{C}$
$$||z| - |w|| \leq |z + w|$$

2. Prove that (a) $2^n < n!$ for $n \geq 4$
(b) $n < 2^n$ for $n \geq 1$.

3. Using only the definition of a limit, prove that
$$\lim_{n \to \infty} \frac{c}{n} = 0$$
for any nonzero $c \in \mathbb{R}$.

4. Prove that the sequence $a_n = 3n$ does not converge to any number $L \in \mathbb{R}$.

5. Suppose $\lim_{n \to \infty} a_n = \alpha$ and that $\alpha > 0$. Prove that $\exists N \in \mathbb{N}$ so that $a_n > 0$ for all $n \geq N$.