

# MATH 347 Homework #10 Due 11/30/07 (in class)

Note Title

#1 a) Prove that if  $f: A \rightarrow B$  and  $g: C \rightarrow D$  are onto, then so is  $h: A \times C \rightarrow B \times D$ ,  $h(a, b) = (f(a), g(b))$ .

b) Prove that if  $f: A \rightarrow B$  and  $g: C \rightarrow D$  are 1-1, then so is  $h: A \times C \rightarrow B \times D$ ,  $h(a, b) = (f(a), g(b))$ .

#2 a) Let  $f: A \rightarrow B$  be a bijection and let  $f^{-1}: B \rightarrow A$  its inverse. Prove that  $f^{-1}$  is a bijection. What is the inverse of  $f^{-1}$ ?

b) Prove that if  $f: A \rightarrow B$  and  $g: B \rightarrow C$  are invertible, then so is  $g \circ f: A \rightarrow C$ . Prove that  $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ .

#3 Prove that the set of irrational numbers is not countable.

#4 Is the map  $\varphi: \mathbb{Z} \times \mathbb{N} \rightarrow \mathbb{Q}$ ,  $\varphi(p, q) = p/q$  1-1? Is it onto? Prove your answers.

#5 Is the map  $f: (0, \infty) \rightarrow \mathbb{R}$ ,  $f(x) = \ln x$  1-1? Onto? Prove your answers.

