I expect you to be familiar with the following topics:

- vectors, dot and cross product
- lines and planes in space
- curves and motion in space
- arclength (this includes being able to parameterize curves!)
- vector fields (14.1)
- line integrals of functions and vector fields (but not the $\int_C P\,dx + Q\,dy$ stuff; we’ll do it later).
- functions of several variables, limits and continuity
- differentiability and partial derivatives
- local maximum and minimum, continuous functions on compact sets achieve max and min
- approximations using first order partials
- chain rule (the whole section except for the implicit function theorem)
- directional derivatives; properties of the gradient.

**Practice test for the first midterm exam, Math 241**

Here are a few problems of the difficulty you may see on the test. The actual exam will have 4-7 problems.

0 True/False questions

T / F If $\vec{a} \cdot \vec{b} = 0$ then $\vec{a}$ and $\vec{b}$ are parallel to each other.

T / F If $\vec{a} \times \vec{b} = \vec{0}$ then $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

T / F The plane $2x - 3y + 5z = 6$ passes through the point (2, 1, 1).

T / F The plane $2x - 3y + 5z = 6$ and the line $\langle x, y, z \rangle = \langle 2t + 1, 2 - 3t, 5t \rangle$ have a point in common.

T / F If $\vec{a} \times \vec{b} = \vec{0}$ then $|\vec{a} \cdot \vec{b}| = |\vec{a}||\vec{b}|$.

T / F All the level curves of the function $f(x, y) = x^2 - y^2$ are straight lines.

T / F If for nonzero vectors $\vec{a}$ and $\vec{b}$ we have $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$ then the angle between $\vec{a}$ and $\vec{b}$ is $\pi/4$.

T / F If $\vec{a}$ and $\vec{b}$ are parallel then $\vec{a} = \lambda \vec{b}$ for some scalar $\lambda$.

1 Write down an equation of the plane through the point (1, 2, -3) normal to the line through the points (0, 1, -1) and (1, 0, 1).

2 Suppose $f$ is a differentiable function with $f(1, 1) = 0$ and $\nabla f(1, 1) = \langle 1, -2 \rangle$. What is $f(1.02, 0.99)$, approximately?

3 The acceleration of a particle in space is $\vec{a}(t) = \langle 0, -\cos t, -\sin t \rangle$. What is the position vector $\vec{r}(t)$ at time $t$ if $\vec{r}(\pi) = \langle 0, -1, 0 \rangle$ and $\vec{v}(\pi) = \langle 1, 0, -1 \rangle$?
4. What are the maximum and minimum values (if any) of the function \( f(x, y) = x^2 - 2y \) in the region \( R = \{(x, y) \mid x^2 + y^2 \leq 1\} \)? Draw a picture of the level curves of \( f \).

5. Suppose the curve \( C \) is parameterized by \( \langle t, t^2 \rangle \), \( 0 \leq t \leq 1 \), \( f(x, y) = x \), \( \vec{F}(x, y) = (-y, x) \). What are \( \int_C f \, ds \) and \( \int_C \vec{F} \cdot d\vec{s} \)?

6. The surface \( x^2 + 4y^2 + 9z^2 = 14 \) is a level surface of a function \( f(x, y, z) \) (I am not telling you what that function is, only that the above ellipsoid is its level surface). Can \( \nabla f(1, 1, 1) = \langle 3, 12, 27 \rangle \)? Explain.

7. If \( z = y + f(x^2 - y^2) \), where \( f(u) \) is some differentialbe function, show that
   \[
   \frac{\partial z}{\partial x} + \frac{x}{\partial y} = x.
   \]

8. The acceleration \( \vec{a}(t) \) of a particle moving in space is \( \langle 2, 0, 6t \rangle \). Find the velocity \( \vec{v}(t) \) and the position \( \vec{r}(t) \) of the particle at time \( t \) if at \( t = 1 \) the position \( \vec{r}(1) = \langle 1, 3, 1 \rangle \) and the velocity \( \vec{v}(1) = \langle 2, 2, 3 \rangle \).

9. Find the arc length of the curve \( \vec{r}(t) = \langle \cos(e^t), \sin(e^t), e^t \rangle \), \( 0 \leq t \leq 1 \).

10. Suppose \( \lim_{(x, y) \to (0, 0)} f(x, y) = 0 \) if \( (x, y) \to (0, 0) \) along the line \( y = x \) and it is 1 if \( (x, y) \to (0, 0) \) along the line \( y = -x \). Then \( \lim_{(x, y) \to (0, 0)} f(x, y) \)
    
    1. exists and equals 0
    2. exists and equals 1/2
    3. exists and equals 1
    4. exists but its value cannot be determined from the information given
    5. does not exist

11. If \( f(u, v) = u^2v \) where \( u = u(t) \) and \( v = v(t) \) are differentiable functions of \( t \) with
    
    \[
    u(1) = 1, \quad v(1) = 1, \quad \frac{du}{dt}(1) = 5, \quad \frac{dv}{dt}(1) = -1,
    \]
    then, for \( t = 1 \), \( \frac{df}{dt} = \)
    
    a. 10  b. -1  c. 9  d. 8  e. 6

12. The plane tangent to the surface \( 2x^2 + xy^2 + z^3 = 2 \) at the point \( (-1, 1, 1) \) is
    
    1. \(-3(x + 1) - 2(y - 1) + 3(z - 1) = 0\)
    2. \(-x - 1 + \langle y + 1 + (z + 1) = 0\)
    3. \(5(x + 1) + 2(y - 1) + 3(z - 1) = 0\)
    4. \(3(x - 1) + 2(y - 1) - 3(z + 1) = 0\)
    5. \(-3(x - 1) - 2(y + 1) + 3(z + 1) = 0\)

13. Consider the function \( f(x, y, z) = 2x^2 + xy^2 + z^3 \). The direction in which the function increases the fastest at the point \( (-1, 1, 1) \) is
    
    1. \( < -3/\sqrt{22}, -2/\sqrt{22}, 3/\sqrt{22} > \)
    2. \( < -3, -2, 3 > \)
    3. \( < 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3} > \)
    4. \( < -1, 1, 1 > \)
    5. \( < 5, 1, 1 > \).