1) Do problems 3.28, 3.44, 3.56, 4.5, 4.9, 4.12, 4.21 from the book.

2) (problem of A.J. Hildebrand) The Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, ... defined by $F_1 = 1$, $F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$ for $n \geq 2$, is one of the most famous mathematical sequences. It has many remarkable properties, pops up all over the place in nature (google fibonacci numbers and pine cones, for example), and is a near endless source of amazing formulas. These formulas are often hard to discover, but routine to prove, using strong induction. The exercises below are intended to practice such proofs.

a) Show that $\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$ for all $n \in \mathbb{N}$.

b) In this problem we prove a formula for $F_n$. Show that $F_n = \frac{1}{\sqrt{5}}(\alpha^n - \beta^n)$, where $\alpha = (1 + \sqrt{5})/2$ (this number is called the golden ratio) and $\beta = (1 - \sqrt{5})/2$. (Hint: use strong induction and the properties that $\alpha^2 = \alpha + 1$ and $\beta^2 = \beta + 1$, which you should first check)

3) How difficult was this homework? How long did it take?