Math 250A: Reading and Concepts for 9/26-10/5

General reading note: We are now transitioning into the study of fields: specifically, roots of polynomials, algebraic/transcendental extensions, finite fields, and Galois theory to name some topics. I will assume that you are very familiar with the following concepts/results from the get-go (F always denotes a field):

- fields, field characteristic, field extension, simple field extension, degree of a field extension, finite extensions, algebraic vs. transcendental elements, minimal polynomial of an algebraic element,
- $F(\alpha) \cong F[x]/(p(x))$ where $p(x)$ is an irreducible polynomial in $F[x]$ and α is a root of $p(x)$; this tells us how to construct a field containing a zero of a given polynomial over F
- Irreducibility criteria for polynomials: see section IV.3 of Lang
- A polynomial $p(x) \in F[x]$ can have at most $\deg(p)$ zeros in F
- A field has characteristic 0 or p for some prime p
- A finite multiplicative subgroup of a field is cyclic: see section IV.1 of Lang
- α is algebraic over F if and only if the degree of α over F is finite
- Finite extensions are algebraic
- Tower theorem: If E is a finite extension of F and F is a finite extension of K then

$$[E : K] = [E : F][F : K]$$

The lectures this week will be planned roughly as follows:

- 9/26: Finishing up with rings and Mason-Stothers theorem. Beginning fields: some motivation for what we will be doing in the next few weeks, transcendental vs. algebraic elements, transcendence degree. Reading: V.1 and VIII.1 of Lang. Concepts you should know: see above.
- 9/28: Finishing discussion of transcendence degree, some properties of composite fields (called a compositum in Lang). Reading: VIII.1, V.1 of Lang.
- 10/1: Splitting fields and algebraic closure. Reading: V.2, V.3 of Lang.
- 10/3: Proof of existence and uniqueness up to isomorphism of algebraic closure, possibly starting on separable extensions. Reading: V.2 (Theorem 2.5), V.4 of Lang. Also see IV.1 on when a polynomial $f(x)$ has multiple roots. It would be useful to read up on Zorn's lemma.
- 10/5: Finishing discussion of separability, classifying finite fields. Reading: V.4, V.5.